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ARKS

THE PROCEEDINGS OF THE PHYSICAL SOCIETY

VOL. 44, PART 4 *July* 1, 1932 NO. 244

THE MEASUREMENT OF REFLECTION COEFFICIENTS FOR OBLIQUE INCIDENCE

BY H. E. BECKETT, B.Sc., Building Research Station

Received February 18, 1932. *Read April* 15, 1932.

ABSTRACT. A method is described whereby the effect of obliquity of incidence upon the reflection coefficients of certain materials can be examined. The material is spread upon the metal receiver of a thermopile which, when exposed to radiation, yields an e.m.f. proportional to the absorption coefficient of the surface. Results obtained with black and white paints and with polished copper are discussed.

§ 1. INTRODUCTION

In a previous paper* a determination of the reflection coefficients of a number of rough surfaces with the aid of a hemispherical integrating mirror has been described. The apparatus used in that investigation was designed to allow the incident beam of radiation to fall normally upon the specimen and cannot readily be adapted for measurements at oblique incidences.

§ 2. METHOD

With certain materials, notably those which can be spread as a thin coating upon a metal sheet, the effect of obliquity of incidence can be investigated in the following simple manner. The material is painted upon the metallic receiver of a thermopile mounted so as to be rotatable about a vertical axis lying in the plane of and bisecting the receiver. With a pencil of radiation falling upon the test surface the deflection of a galvanometer connected to the thermopile is proportional to the fraction of the incident energy absorbed, and will in general vary as the surface is rotated. The information obtainable in this way, together with an absolute measurement of the reflection coefficient of the surface for normal incidence with the same type of radiation, enables the variation of its reflection coefficient to be examined over a wide range of incidence.

* H. E. Beckett, *Proc. Phys. Soc.* 43, 227 (1931).

§ 3. APPARATUS

A convenient form of thermopile, whose single thermo-electric element is constructed with fine wires (46 s.w.g.) of copper and constantan, is shown in figure 1. Parallel constantan wires stretched across an ebonite frame support two receivers of copper foil, at the centres of which the copper leads are attached.

Both receivers are coated with the test material and, being of equal size (2 cm. × 1 cm. × 0·1 mm.), are similarly controlled by the radiative and convective effects of their surroundings. An extremely stable zero is thus assured. It has been shown experimentally that, as a result of the high thermal conductivity of the copper foil, there is no appreciable variation in sensitivity over the surface of the receiver.

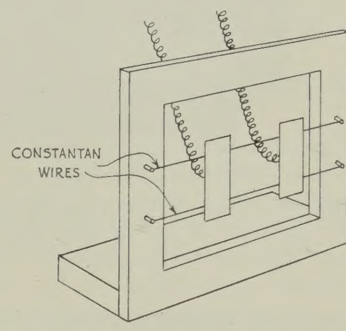

Fig. 1. Thermopile for reflection tests.

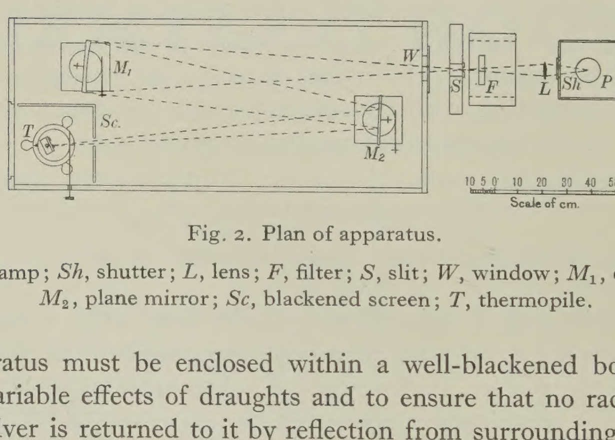

Fig. 2. Plan of apparatus.

P, pointolite lamp; *Sh*, shutter; *L*, lens; *F*, filter; *S*, slit; *W*, window; M_1, concave mirror; M_2, plane mirror; *Sc*, blackened screen; *T*, thermopile.

The apparatus must be enclosed within a well-blackened box to avoid the exceedingly variable effects of draughts and to ensure that no radiation reflected from the receiver is returned to it by reflection from surrounding surfaces. With the thermopile mounted upon a worm-driven turn-table, rotation can be effected by means of a key inserted through a small hole in the side of the box, while the angular position of the turn-table can be read through a window.

The radiation is conveniently produced and controlled by means of the apparatus described previously. Tests can then be made with the artificial sun (Pointolite and gold filter) or at various wave-lengths determined by the filters adopted. It is

essential that the image projected upon the test surface should be in the form of a narrow rectangle, so that even with large angles of incidence the entire image is intercepted by the test surface. With an image measuring 0·8 mm. in width the limiting angle of incidence is 85°.

The general lay-out of the apparatus is shown in figure 2.

§ 4. CALIBRATION OF RADIATION-RECORDER

Apparatus of the above type has been employed in the calibration of the spherical receiving surfaces of an instrument designed by Mr Dufton of the Building Research Station for recording the intensity of solar radiation. Such instruments are commonly provided with receiving surfaces lying in a horizontal plane and, with a low sun, must give very erroneous readings owing to the rapid increase of the reflection coefficient of a blackened surface as grazing incidence is approached. Mr Dufton's instrument embodies two spherical receiving surfaces, painted black and white, whose effective reflection coefficients should be independent of the height of the sun.

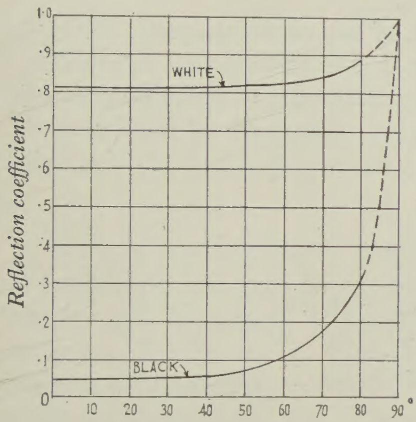

Fig. 3. Reflection coefficients of black and white painted surfaces.

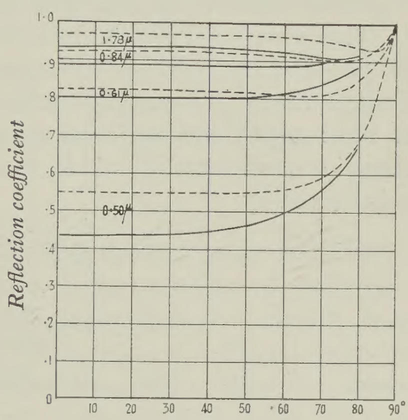

Fig. 4. Reflection coefficients of polished copper.

It may be shown that the energy of radiation absorbed by a sphere of radius r is equal to r

$$\pi r^2 I \int_0^{\pi/2} (1 - R_\theta) . \sin 2\theta . d\theta,$$

where R_θ is the reflection coefficient of the surface for the angle of incidence θ, R_θ, θ
and I is the intensity of radiation. Since πr^2 is the projected area of the sphere, I
the value of the integral may be termed the effective absorption coefficient of the spherical surface.

With the artificial sunlight of the Pointolite and gold film combination the reflection coefficients at normal incidence of the black and white cellulose paints used in the sunshine recorder were found to be 0·051 and 0·813 respectively.

The variation of the reflection coefficients with obliquity of incidence is shown in figure 3. Each point represents the mean of observations on each side of normal

incidence, so that any slight inaccuracy in the assumed position of normal incidence should not lead to appreciable error. From the curves the effective absorption coefficients of the black and white spherical surfaces have been computed as 0·899 and 0·175 respectively.

§ 5. OBSERVATIONS ON POLISHED COPPER

In view of the considerable theoretical interest attaching to metallic reflection, the effect of obliquity of incidence upon the reflection coefficient of polished copper has been investigated. For this purpose suitable bands of radiation were separated by means of filters from the spectrum of a Pointolite lamp, and were allowed to fall directly on the polished copper receiver of the thermopile.

The results obtained are shown in the full curves of figure 4, the means of readings on both sides of normal incidence being again used. One characteristic should be noted; at the longer wave-lengths a dip in the reflection curves is found which becomes more pronounced and at the same time shifts towards the angle of grazing incidence as the wave-length increases.

This is to be expected from the application of the electromagnetic theory to the problem, the results of which are quoted in convenient form in the *International Critical Tables**. A metallic surface behaves very differently to radiation polarized at right angles and radiation polarized parallel to the plane of incidence, the reflection coefficients $R_\perp$ and $R_\parallel$ of the surface for the two directions of polarization being given by

$R_\perp$, $R_\parallel$

$$R_\perp = \frac{(m_\theta - \cos\theta)^2 + m_\theta^2 . k_\theta^2}{(m_\theta + \cos\theta)^2 + m_\theta^2 . k_\theta^2}$$

and

$$R_\parallel = R_\perp . \frac{(m_\theta - \sin\theta . \tan\theta)^2 + m_\theta^2 . k_\theta^2}{(m_\theta + \sin\theta . \tan\theta)^2 + m_\theta^2 . k_\theta^2},$$

m_θ where

$$2m_\theta^2 = [(n^2 - n^2k^2 - \sin^2\theta)^2 + 4n^4k^2]^{\frac{1}{2}} + (n^2 - n^2k^2 - \sin^2\theta)$$

and

$$2m_\theta^2 . k_\theta^2 = [(n^2 - n^2k^2 - \sin^2\theta)^2 + 4n^4k^2]^{\frac{1}{2}} - (n^2 - n^2k^2 - \sin^2\theta)$$

and θ is the angle of incidence;

n_θ — n_θ the index of refraction for angle of incidence θ;

n — n the index of refraction for normal incidence;

k_θ — k_θ the index of absorption for angle of incidence θ; and

k — k the index of absorption for normal incidence.

From the above expressions it is found that whereas the reflection coefficient $R_\perp$ rises smoothly from its value at normal incidence to the value 1·0 at grazing incidence, the coefficient $R_\parallel$ first falls to a minimum value and then rises steeply to the value 1·0. The reflection coefficient of the surface for unpolarized incident light is $\frac{1}{2}(R_\perp + R_\parallel)$ and the graph obtained by plotting this quantity against angle of incidence may or may not show a dip according to the values of the constants n and k.

* *International Critical Tables*, 5, 248.

At wave-lengths in and near the visible spectrum it is not practicable to determine n and k for a metal except by a reverse application of the theoretical expressions. Usually these constants are computed from observations of the ellipticity of polarization of light reflected from the metal surface at various angles of incidence.

A number of such determinations for copper are quoted in the *International Critical Tables*, and it is of interest to compare the reflection coefficients which can be derived from them with the present measurements. The constants n and k vary somewhat according to the manner of preparation of the copper specimen, but the values given in table 1 seem to be fairly representative of copper which is thick enough to be opaque. The wave-lengths chosen are those about which the energy transmitted by the various filters used in the present investigation is equally shared.

With these values the variation of the reflection coefficient with angle of incidence is shown in the dotted curves of figure 4. The curves show the same general characteristics as those determined by the thermopile method, although the quantitative agreement is poor.

Table 1. Optical constants of polished copper

Wave-length (μ)	n	k
0·50	1·17	2·03
0·61	0·56	5·65
0·84	4·35	10·4
1·78	0·73	13·0

Some of the difference is certainly due to the finite spectral width of the bands of radiation transmitted by the filters. In order to compute reflection coefficients which shall be strictly comparable with those measured experimentally, it is necessary to take into account the spectral distribution of the filtered radiation and the variation of n and k with wave-length. The data given in the *International Critical Tables* enable this to be done at the two shorter wave-lengths, and it is found that for normal incidence the correction lowers the computed coefficients appreciably, in one case below that found experimentally (see table 2). At the two longer wave-lengths the correction can only be small, since the reflection coefficient of copper does not vary rapidly in this part of the spectrum.

Table 2. Reflection coefficients of polished copper at normal incidence

Filter		Reflection coefficients			
			Computed from n and k		
No.	Nominal wave-length (μ)	Measured	At nominal wave-lengths	Corrected for spectral distribution	Standard, corrected for spectral distribution
I	1·78	0·94	0·969	—	0·943
II	0·84	0·89	0·922	—	0·886
III	0·61	0·80	0·821	0·782	0·699
IV	0·50	0·43	0·548	0·529	0·418

Even so there remain appreciable differences between the measured and computed values, which would appear to indicate that the standard values of n and k do not hold accurately for the specimen of copper used. Unfortunately facilities for checking these constants by the ordinary methods have not been available.

Tool* has stated that n and k can be affected by the presence of films of impurity derived from commercial metal polishes, one of which was used in the present experiments. The negligible smallness of such an effect has, however, been shown by further reflection tests with rouge-polished specimens, which have yielded coefficients almost identical with those previously found.

In conclusion it is noted that the standard reflection data for copper quoted in the *International Critical Tables* (see table 2) show considerably better general agreement with the present measurements than with the computed figures.

* A. Q. Tool, *Phys. Rev.* **31**, 1 (1910).

A DIRECT-READING γ-RAY ELECTROSCOPE

By L. G. GRIMMETT, B.Sc., Assistant Physicist,
Westminster Hospital, London

Communicated by B. L. Worsnop, March 14, 1932. Read and discussed April 15, 1932.

ABSTRACT. A dead-beat direct-reading γ-ray electroscope having a linear scale graduated in milligrams of radium is described. It is a combination of a special ionization chamber, Lindemann electrometer and high resistance, and allows the estimation, in less than three seconds, of the activity of small γ-ray sources of the order 1 mgm. Ra with an accuracy of $\frac{1}{2}$ per cent on a full scale deflection.

§1. INTRODUCTION

This instrument was designed for the rapid estimation of the γ-ray activity of the small radio-active appliances, usually called "radon seeds*," which are employed therapeutically in large numbers in hospitals and radium institutions.

A high degree of accuracy is not called for in an instrument for this purpose; the chief requirement is speed of operation, so that the time of measurement and exposure of the operator may be reduced to a minimum.

The well-known circuit of figure 1 has been applied in the convenient form shown in figure 2: the ionization current set up in a chamber C of special design is passed through a high resistance R, and the steady potential difference between the ends of the resistance is measured by means of a Lindemann electrometer L. When certain precautions are taken, the deflection of the electrometer needle is proportional to the γ-ray activity of the source, and if the constants of the instrument are suitably adjusted the deflection can be read off directly in millicuries or milligrams of radium on the eyepiece scale of the observing microscope.

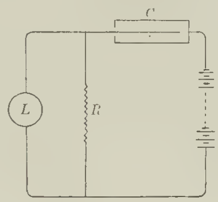

Fig. 1. Circuit.

The Lindemann electrometer is particularly suitable for measurements of this kind, because it is dead-beat and has a high sensitivity, a short period, a linear scale and a stable zero.

* Radon seeds consist of small tubes of gold, platinum, or silver, into which are inserted fine sealed glass capillaries containing a few millicuries of radium emanation. The emanation and its active deposit emit α-, β-, and γ-rays. The α-rays and most of the β-rays are absorbed by the glass and metal containers, so that a seed furnishes a compact γ-ray source, the activity of which decays at approximately the same rate as that of radon, the half-value period being 3·86 days. A typical radon seed would be a platinum tube 5 or 6 mm. long, 1·5 mm. in diameter, with a wall-thickness of 0·5 mm., and containing from 0·5 to 2·5 millicuries of radon.

§2. DESIGN OF THE IONIZATION CHAMBER

In order to avoid the difficulties associated with the use of very high resistances, it is desirable to work with currents as large as possible. Experiments were therefore made to find out what sort of chamber best utilized the feeble ionizing power of the small γ-ray sources concerned. So far, the investigation has been confined to air chambers at atmospheric pressure, because the use of heavy gases for increasing the currents was considered unsuitable for routine work.

Whatever the shape of the chamber, the currents must be approximately saturated and the insulated electrometer system must be carefully protected against ionization leakage to surrounding earthed conductors.

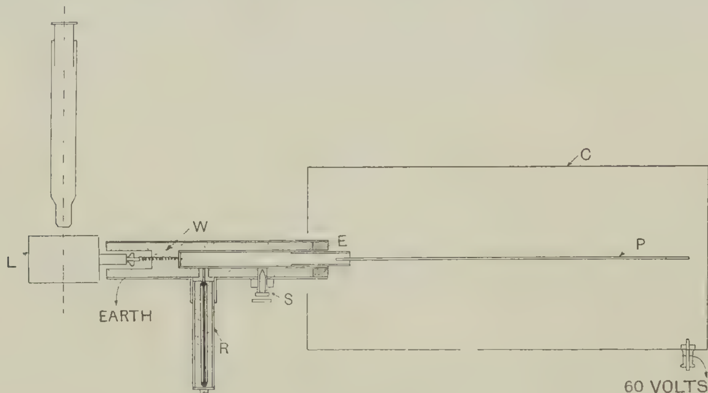

Fig. 2. Constructional details.

C, metal casing; *P*, plate electrode; *L*, Lindemann electrometer; *R*, high resistance; *W*, paraffin wax; *S*, spring earthing plunger; *E*, ebonite collar.

Losses of the latter kind are avoided in the apparatus shown in figure 2: an insulated metal plate *P*, totally enclosed by a charged metal casing *C*, is connected to the electrometer by a lead which passes down the centre of an earthed tube filled with paraffin wax; the latter is recessed at its outer end so as to enclose the electrometer terminal. The whole arrangement was tested for ionization leakage by measuring the rate of increase of potential of the insulated system, with the high resistance removed, when exposed to a weak γ-radiation. The absence of ionization losses is proved by the linearity of the curve so obtained (full line, figure 3) which shows that the current to the electrometer is constant and independent of the potential of the system. When leakage is present a falling-off of current is observed at the higher potentials (dotted line, figure 3). Under the latter conditions, when the high resistance is replaced, the current flowing through it will always be less than that arising in the chamber itself, and no proportionality can be expected between the potential across the ends of the resistance and the γ-ray activity of the source.

When a chamber satisfactory in this respect had been obtained, a properly shielded high resistance of suitable magnitude was made up in the manner detailed

below, and fitted to the lead-in tube, figure 2. Thereafter this part of the circuit was not changed, so that the currents obtained from different chambers under various conditions could be conveniently compared by observing the steady deflection of the electrometer needle.

First of all, the saturation voltage was determined for two rectangular chambers measuring respectively 21·5 × 23 × 12 cm. and 22 × 23 × 24 cm., each provided with a rectangular inner electrode whose dimensions were a little smaller than those of the outer casing. The size of the inner plate was found to be unimportant,

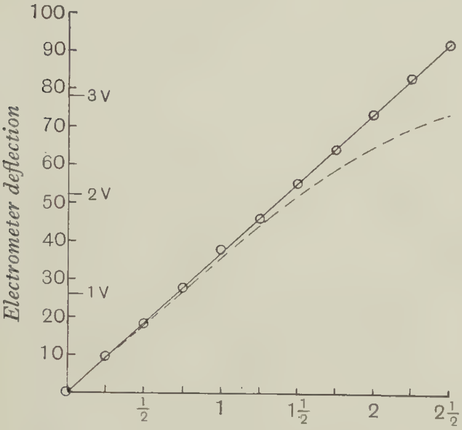

Fig. 3. Test for presence of ionization leakage.

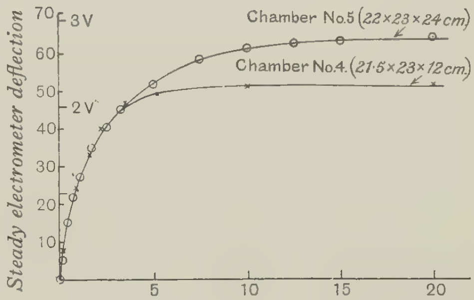

Fig. 4.

as might be anticipated, since all the ions generated in the air space are swept to the plate. The γ-ray source, a platinum needle 25 mm. long and containing 5 mg. of radium, was placed where it gave the strongest ionization current, viz. in the centre of the top of the metal casing *C*. As will be seen from figure 4, a field of 10 to 15 V./cm. brings the currents sufficiently near to saturation for readings proportional to the strengths of the sources to be obtained.

The influence of the volume of the chamber on the magnitude of the saturation currents was next investigated by fitting rectangular chambers of different sizes. In each case the 5-mg. needle was placed on the outer casing, in the centre. Figure 5 and the table show how the saturation current depends upon the dimensions of the chamber.

Table. Variation of saturation current with dimensions of chamber

No. of chamber	Dimensions* (cm.)	Volume ($cm.^3$)	Relative saturation currents (scale divisions)
1	13 × 11 × 12	1,720	22
2	21·5 × 23 × 6	2,960	36
3	16 × 17 × 12	3,260	33
4	21·5 × 23 × 12	5,920	39·5
5	23 × 22 × 24	12,200	46

* The last figure gives the height of the chamber.

Evidently there is not much to be gained by using a volume greater than 5 or 6 litres.

Now in all of the above measurements only half of the radiations issuing from the radioactive substance actually passed through the ionization chamber, and an obvious way of augmenting the current is to place the source inside the chamber so that all its radiations are utilized. The change was found to double the current approximately, but in practice this procedure is inconvenient and has also the disadvantage that it restricts the use of the instrument to the measurement of pure γ-ray sources.

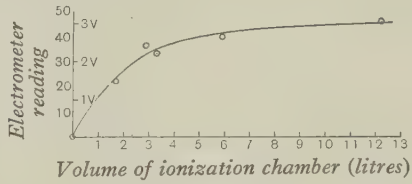

Fig. 5. Variation of saturation current with volume of rectangular ionization chamber.

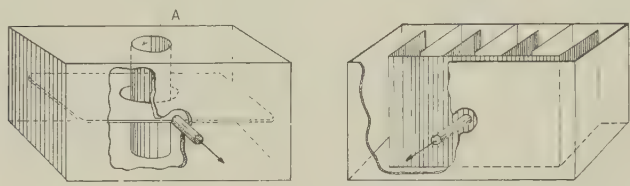

Fig. 6.

A type of chamber which is free from these defects and yet makes use of nearly all the rays emitted by the source is shown in figure 6 (*a*). The radium needle was placed half-way down the axis of the tube *A*, which had a diameter of 4 cm.

With this arrangement, the saturation current was only slightly greater than that obtained from a simple rectangular chamber of the same dimensions with the same radium needle on the outer casing; it would appear from this and the foregoing experiments that the bulk of the ionization effects arise in the immediate vicinity of the source.

This particular form of chamber was found, however, to possess the useful property that the currents obtained from it were practically independent of the position of the source within the limits of a cylindrical zone (about 4 cm. high and 1 cm. in diameter) in the middle of the tube *A*; consequently the activity of all sources, whatever their size and distribution, can be directly compared, so long as they can be contained by a cylinder of these dimensions. The small gain in current obtained with this pattern did not warrant its adoption for ordinary work, and its use was therefore confined to the comparison of sources of unequal size. Further efforts were made to improve on the simple closed box type.

An increase of current was sought by making use of the well-known fact that a large proportion of the ionization is due to photo-ions liberated from the metallic

walls of the chamber. Successful attempts were made to increase the photoelectric emission in two days, firstly by fixing a multiplate electrode system, figure 6 (*b*), designed to present a large surface to the radiations, and secondly by lining the chamber with different metals.

The saturation currents recorded with the multiplate electrode were about 20 per cent higher than with a single-plate electrode, and the effect of partially lining the chamber with aluminium, brass and lead was to give currents in the ratio 38 : 42 : 46. Thus as far as can be seen from these experiments on air chambers at atmospheric pressure, the best results may be expected from a lead-lined chamber having a volume of 5 or 6 litres, and provided with a multiplate electrode of some heavy metal.

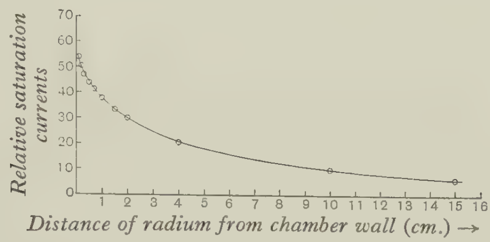

Fig. 7. Variation of current with position of radium.

The maximum γ-ray ionization current yielded by such an arrangement is roughly 5×10^{-11} A. per mg. of radium, although this value may be reduced somewhat by the filters necessary to cut off β-radiation from unscreened radon tubes.

The variation in the electrometer reading as the radium is moved about on the top of the casing of a simple rectangular chamber is remarkably small. In fact, within a radius of 2 cm. from the centre no difference at all can be detected. There is, however, a marked reduction of current as the source recedes from the chamber (figure 7).

§ 3. CONSTRUCTION OF THE HIGH RESISTANCE

For the purpose of this instrument a resistance of the order $10^{10}\ \Omega$ is required. A satisfactory graphite leak can be constructed in the following manner: a heavy pencil line is drawn between two terminals on a strip of some hard highly insulating material such as orca or ambroid. The graphite streak is polished vigorously with cotton wool until most of it has been removed; the resistance is tested from time to time by connecting it to the ionization chamber, and the polishing process continued until a suitable value is obtained. Formed in this way, the graphite layer is firm and durable and is not easily damaged. Finally the resistance unit is mounted in a metal tube and sealed up with paraffin wax, which serves the double purpose of eliminating ionization losses and also of protecting the graphite film from the atmosphere. The waxing process sometimes alters the value of the resistance slightly, especially if the wax be poured in too hot.

§4. GENERAL REMARKS ON THE USE OF THE INSTRUMENT

The instrument is calibrated by laying a γ-ray source of known value on the top of the chamber and adjusting the sensitivity of the electrometer until the steady deflection of the needle corresponds to the numerical strength of the radium standard. The activity of other γ-ray sources of similar filtration placed in the same position can then be read directly in milligrams or millicuries on the eyepiece scale. An accuracy of about $\frac{1}{2}$ per cent on a full scale deflection can be obtained if the calibration process is repeated at frequent intervals.

Sources of different diameters may be mounted in a V-shaped trough at the side, so that their centres are at the same distance from the wall of the chamber, or they may be compared by using the special chamber of figure 6 (*a*).

To expedite the measurement of radon seeds, for which this apparatus was primarily intended, the screening of the radium standard and its distance from the chamber are simulated, in the case of naked radon capillaries, by the insertion of an aluminium disc of a thickness determined by trial. Discs of other thicknesses are used in the measurement of radon seeds having a filtration different from that of the radium standard. Screening corrections are automatically eliminated by this device.

It should be noted that the group reading of a number of radon seeds will be less than the sum of the individual strengths, unless the seeds are set sufficiently far apart (about 5 mm.) for the mutual screening of the oblique rays to be negligible.

The range of measurement can be extended by varying the electrometer sensitivity, by changing the value of the high resistance and by altering the position of the radio-active source. The effects of stray γ-radiation can be nullified by displacing the zero of the scale so that no corrections are needed for measurements carried out near other radium. Adequate protection for the operator can be provided, for since the instrument functions on current, the lead-in tube can be made as long as desired without loss of sensitivity.

Further experiments are at present in progress on high-pressure air-chambers, the results of which will be published in the *Journal of the British Institute of Radiology*.

DISCUSSION

Prof. F. L. Hopwood emphasized the remark made at the end of the paper as to the importance of protecting the operator. In the absence of suitable protection the latter would be constantly exposed to radiation which, though its intensity might be too small to produce early effects, would be very harmful if endured during a long period.

Prof. A. O. Rankine suggested that the necessity for frequent calibration with a sub-standard comprising a known γ-ray source could be avoided by the use of a standardized e.m.f.

Mr R. S. Whipple suggested that a potentiometer might be incorporated in the instrument for the purpose mentioned by the last speaker.

Author's reply. I am not sure whether the high resistance remains sufficiently constant for the method suggested by the last two speakers to be reliable.

STUDIES IN INTERFEROMETRY
(1) A NEW TYPE OF INTERFERENCE REFRACTOMETER

By W. EWART WILLIAMS, M.Sc., Lecturer in Physics,
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ABSTRACT. The importance of accurate refractive-index measurements in relation to spectroscopy is emphasized and the various methods hitherto employed are compared. A new type of interferometer is described in which the beam from the central part of the collimator objective is divided and each half laterally displaced. After passing through the gas tubes, the beams are again reunited so that the interference pattern at the focal plane of the telescope objective is similar to that from a two-plate transmission échelon. The brightness of the fringes is approximately twenty times greater than with the usual Rayleigh interferometer, so that the instrument can be combined with a spectrograph for dispersion measurements. A new method, based on Benoît's "fractional-part" principle, is developed for this purpose.

§ 1. INTRODUCTION

The accurate determination of the refractive indices of gases has become very important in spectroscopy. Within recent years, very considerable progress has been made in the standard of accuracy within which the wave-length of a spectral line can be determined in terms of that of the standard red cadmium line. This increased accuracy is in part due to improved technique and knowledge as to the best methods of exciting the source, and in part to improved apparatus and methods for effecting the intercomparison. For example, the wave-lengths of some of the sharp satellites in the line spectra of certain heavy elements can be determined to within one or two units in the fourth decimal place in Ångstroms, yet even when such a line is close to that of red cadmium so that the conditions of pressure and temperature during exposure are of little account, an uncertainty of only one unit in the seventh decimal place of the refractive index of dry air at 15° C. and 760 mm. pressure may introduce an error of six units in the fourth place in Ångstroms in the vacuum wave-length required for theoretical calculations. It is fairly safe to say that the probable error in the tables of the refractive index of air in the range of 2000 to 8000 Å. is at least three or four units in the seventh place, so that an experimental accuracy of $\pm$ 0·001 Å. at 6000 Å. will involve an uncertainty of $\pm$ 0·002 Å. when a conversion to vacuum wave-length is made. In the ultra-violet, the uncertainty measured in Ångstroms will be proportionally smaller, but the uncertainty in frequency or wave-number (which is the quantity required for theoretical work) becomes much greater.

§2. METHODS

The various interferometric methods that have been employed to determine the refractive indices of gases divide themselves sharply into one of two classes: (A) Methods in which the total path-difference involved is due to change of refractive index obtained by exhausting or filling one tube; and (B) Methods whereby only a change in an already large path-difference is determined, corresponding to the change of the refractive index from its normal value to unity as the vessel is exhausted.

The various methods in which a Fresnel bi-prism or the Jamin or Rayleigh interferometers are used form examples of the (A) class. Rentschler's* method of using a Fabry-Perot étalon is the only example of the (B) class that has hitherto been used, although other methods could be devised. Thus Köster's† application of the Twyman and Green‡ interferometer for wave-length measurements could be easily adapted for refractive index measurements by enclosing the gauge or reference block in a chamber which could be exhausted.

The most complete set of data on the refractive index of air is the now classical work of Meggers and Peters§ at the Bureau of Standards. They employed Rentschler's method, and their tables are those in general use for converting wave-lengths determined in air to the corresponding vacuum values.

The Fabry-Perot method, used in combination with a grating, enables the determination to be made simultaneously on a large number of lines, and it can be extended by suitable means to the near infra-red and ultra-violet. For convenience and rapidity the method could hardly be improved upon, but for obtaining the greatest accuracy it seems fundamentally weak. The refractive index μ is given in this method by the ratio of the orders of interference n_{air} and $n_{\text{vac.}}$ determined by the photographs taken respectively with an air-filled and an exhausted Fabry-Perot interferometer.

It is generally accepted that under very favourable conditions of a very sharp line and a high reflection coefficient for the silver films, the fractional part can be determined to even better than 1 per cent. When, however, the fringes become broad, either because the line is in a region where the reflection coefficient of the films is reduced, or because the actual width of the line itself becomes an appreciable fraction of the range (in Ångstroms) between successive fringes, the accuracy within which the fractional part can be determined is considerably reduced. In Megger's and Peter's experiments the value $(n_{\text{air}} - n_{\text{vac.}})$ varied between 5 and 25 with interferometer-plate-separations varying between 3 and 25 mm. A final accuracy of 1 per cent in the determination of the fractional part means an uncertainty of between 1 and 6 units in the seventh decimal place for μ_{air}. The inherent defect of the method is that if larger plate-separations of the interferometer were

* *Astro. J.* **28**, 345 (1908).

† *Z. f. Feinmech.* **34**, 55 (1926); and see also Weber, *Phys. Z.* **29**, 233 (1928).

‡ *Phil. Mag.* **35**, 49 (1918).

§ *Bull. Bur. Stand.* **14**, 697 (1918).

chosen, there would be a corresponding reduction in the accuracy within which the fractional parts could be determined, so that increased accuracy could only be obtained by using more homogeneous light-sources. While lines of far greater homogeneity than the iron lines are well known, these do not suffice to yield representative values throughout the spectrum. It is therefore desirable to consider the possibilities of methods of the first class, the final accuracy of which do not depend on the homogeneity of the spectral line which is used.

A Jamin interferometer with tubes 100 cm. in length will show a displacement of about 560 fringes with a wave-length of 5000 Å. when one of the tubes is exhausted from atmospheric pressure. If the displacement be measured to within a tenth of a fringe, the possible error in μ is approximately half a unit in the seventh decimal place. Owing to the greater length, the difficulties of temperature- and pressure-measurements are now much greater, but they should not be insuperable since this very much greater accuracy only requires that the initial temperature and pressure be accurately measured to within 0·05° C. and 0·1 mm. respectively. The main difficulty with the Jamin interferometer is that it cannot be conveniently combined with a spectrograph. It is dependent, even in visual work, on an artificial fiducial mark or cross wire in the eyepiece, which would in no way take account of any small instrumental change (such as the tilting of a Jamin block or a small twist of the framework) that might occur during or between the two exposures.

The Rayleigh interferometer has a fiduciary set of fringes formed by the light traversing that part of the interferometer which is not occupied by the tubes. Any accidental disturbance or deformation of the interferometer itself would cause an equal displacement of both sets of fringes. By using the method in which coincidence of the upper and lower fringes is adjusted by means of a calibrated compensation plate, a far higher standard of accuracy can be attained than by estimating the position of a cross-wire between two fringes. It is generally accepted that an accuracy of about $\frac{1}{60}$ to $\frac{1}{40}$ of a fringe can be obtained by means of this coincidence method.

While the Rayleigh interferometer forms the most accurate means of determining the refractivity of gases, it can only be employed with the very strongest monochromatic radiations, such as the lines from a mercury arc. White light fringes can be employed for many industrial applications of the instrument provided the path-difference introduced is not too great, and it is now customary in such cases to use a Pointolite lamp to obtain the fringes with maximum clearness or visibility and comfortable brightness.

In order to obtain dispersion curves, the method of selecting a narrow range of the white-light spectrum has been suggested. This narrow band is then to be passed into the Rayleigh interferometer, and the fringe-displacement is supposed to give the refractive index for the mean wave-length. This method is open to the very serious objection that it may not be the ordinary phase refractive-index μ_0 that is measured but a group index μ_g defined as (velocity in vacuo)/(velocity of group in medium), in which case μ_g

$$\mu_g = \mu_0 \left(1 - \frac{\lambda}{\mu_0} \cdot \frac{\partial \mu_0}{\partial \lambda} \right).$$

The reality of this effect is shown by the abnormal displacement of the fringe system obtained by R. W. Wood* with sodium vapour, when the Zeeman component of the helium line used as the source was drawn into the anomalous region of sodium by a suitable magnetic field. In this region $\partial\mu/\partial\lambda$ was so great that the effect of this term caused a displacement twice as large as that which would be obtained by the phase index alone. The dispersive power of air is so low that $\left(\frac{\lambda}{\mu_0}\frac{\partial\mu}{\partial\lambda}\right)$ is of the order of 0·00001, yet the absolute refractive-index is required to such a high precision that this uncertainty cannot be allowed. The difficulty is to know at what point the band of spectrum transmitted into the Rayleigh instrument can no longer be regarded as monochromatic radiation, so that it is the group velocity or refractive-index that is being measured. Wood's experiment shows that it must depend on the magnitude of $\partial\mu/\partial\lambda$, since in the anomalous region of sodium the relatively monochromatic Zeeman component of the helium line D_3 must be considered as a narrow band of continuous radiation. The writer hopes to carry out some experiments on this problem in the near future.

§3. DESCRIPTION AND EXPLANATION OF NEW METHOD

The fundamental disadvantage of the Rayleigh interference refractometer arises out of the fact that the primary slit of the collimator must be kept very narrow. As this slit is opened, the brightness of the fringe system increases rapidly, but after a certain point the visibility (as defined by Michelson) or clearness rapidly deteriorates until, when the slit subtends at the near nodal point of the collimator lens an angle equal to the angle between the fringes, all traces of a fringe pattern vanish. This is analogous to Fizeau's† method of finding the angular diameters of stars, which has been so successfully developed by Michelson.

W Since the angle between the fringes is given by λ/W, where W is the distance between corresponding points of the double slit, the obvious way of increasing the brightness would be to reduce W to its minimum value. In the standard commercial instrument the width of each double slit is 4 mm. and W is 12 mm. This latter value cannot be substantially reduced since a slight lack of alignment of the long gas tubes would effectively vary W. It is further desirable that the light should pass through the centre of a tube and not near the walls.

It will be remembered that in order to apply Fizeau's method to measure the angular diameter of stars, Michelson‡ employed two outer mirrors, with separation
b, d b, to receive the light from the distant star, and two inner mirrors, with separation d, to project the light beams into the telescope. This enabled him in effect to magnify the star, as far as interference fringes were concerned, in the ratio (b/d); for although the angle between the fringes remained at (λ/d), the fringes disappeared when the
K star subtended an angle $K(\lambda/b)$, K being a constant whose value is 1·22 for a uniformly glowing disc.

* *Phil. Mag.* **8**, 320 (1904).
† *Compt. Rend.* **66**, 934 (1868).
‡ *Astro. J.* **51**, 257 (1920).

The ordinary double-slit arrangement of the Rayleigh instrument, shown for reference in figure 1 (a), is modified as shown in figure 1 (b). A single slit S_b of width $2a$ is used and the beam is divided and displaced by some optical device so that the two beams have the same relative separation W as before. As far as the observing telescope is concerned, the angle between the fringes will still be (λ/W), but the primary slit Pa may now be opened until it subtends an angle $\frac{1}{2} \times 1{\cdot}43\lambda/a$ at the collimator objective before the fringes disappear. The arrangement in fact practically amounts to a Michelson star interferometer used in the reverse way, with the difference that the distance between the outer beams is fixed, and that we are using the arrangement for an entirely different purpose—to increase the brightness of the fringes.

While the same standard of visibility of the fringes is retained, the primary slit can now be opened to a width $0{\cdot}715W/a$ of its previous value, so that the brightness* of the fringes is increased as the square of this ratio. This, with the dimensions of W and a already quoted, should give an increase in brightness of $4{\cdot}6$ times if the losses at the displacing device be neglected.

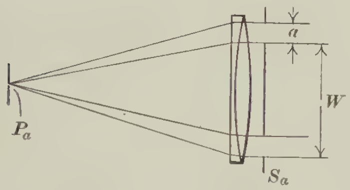

Fig. 1 (a).

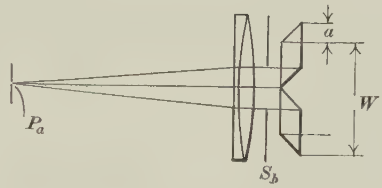

Fig. 1 (b).

The correctness of the above conclusions was first tested through the cooperation of Messrs A. Hilger Ltd. by using small totally-reflecting prisms as shown in figure 1 (b). It was found extremely difficult to work the tiny prisms to the necessarily very high standard required in interferometry. The optical retardation in each had to be identical, and the working of a long parallel strip prior to cutting in two proved impracticable owing to the springy nature of the very thin plate. Finally, it was found difficult to adjust two such small prisms and to hold them without distortion. Hence, even at the expense of a much greater loss of light, the well-known method of Albrecht's rhomb (see figure 2) was employed to separate the two beams. The first trial with an Albrecht rhomb was unsuccessful, and no

* If the image of the source were very carefully focussed by means of a wide-angled condenser lens system on an extremely *thin* slit, the gain in brightness would only be $2{\cdot}145$ times. Such a slit cannot be made. In practice, the usual spectroscope slit can be compared to a narrow opening between long parallel walls so that the oblique pencils from the condenser cannot enter. The light that reaches the objective enters the slit approximately as a parallel beam. When this slit is sufficiently narrow so that the whole objective is filled by the central maximum of this slit, a further reduction in slit-width to a half will reduce the intensity of the final image to a quarter since not only is the flux of light that enters the slit halved, but the light that enters is diffracted over twice the angle and consequently only half of it can enter the aperture. This is the principle of Vierordt's method of photometry, and explains the desirability of spectrographs having collimators of much longer focal length than telescopes or cameras, in which case the slit can relatively be much wider.

fringes could be obtained. By placing a narrow cross slit on the primary slit, the reason of the failure became evident—the point image from one half of the beam was slightly lower than that from the other, showing that the rhomb had a small pyramidal error of a few seconds of arc. When this was remedied, the fringes were easily found when the rhomb was placed symmetrically about the aperture and with its refracting edges parallel to the primary slit. It was verified that the primary slit could be opened two or three times wider than before without loss of visibility of the fringes.

The problem of the gain in intensity or brightness was more difficult, but on comparison of the brightness with various neutral-tint plates in the path of the modified instrument, the fringes had approximately the same brightness as in the original Rayleigh interferometer when the plates reduced the brightness to a quarter or a fifth. The exact value is relatively unimportant, the main point being to prove that the gain in brightness is more than proportional to the increased width of the primary slit.

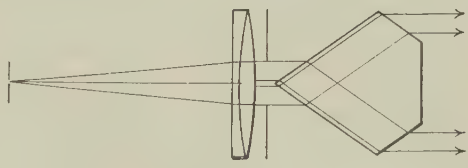

Fig. 2.

Hitherto, the instrument behaves (apart from the increased brightness of the fringes) as a standard Rayleigh interferometer; the angle between the fringes and the distribution of light among the fringes remain exactly as before. The fringes at the focal plane of the telescope objective are so close together that a very short-focal-length cylindrical rod of glass must be used to magnify them sufficiently. This, not being achromatic, requires refocussing in different parts of the spectrum, so that in spite of the increased brightness of the fringes, the modified Rayleigh instrument cannot be successfully combined with a spectrograph.

The success of the first separation of the two beams from the centre naturally led to a trial of the reverse process at the telescope end. When the two beams are recombined by a second rhomb or analogous device, their separation W on entering the telescope objective is reduced to a third. The angle between the fringes increases from $(\lambda/3a)$ to $(0{\cdot}715\lambda/a)$ and the focal length of the eyepiece can be doubled while the same magnification as before is retained. As far as the telescope is concerned, what we now have is a two-plate transmission échelon, and the six or eight fringes obtained with the standard Rayleigh are reduced to the two or three orders seen with an échelon grating. It seems very difficult to derive an expression for the gain in brightness due to the reduction of the number of fringes visible, but practical experiment indicates that the gain on recombination of the beams is even greater than that obtained at the first stage. One contributory factor is that the reflection losses at the second rhomb are now much smaller, since the beams emerging from the first rhomb are, to a considerable extent, polarized.

The nett overall gain of brightness when two rhombs are used as described has not been carefully measured, but a conservative estimate would be about twenty times. The increase in brightness will be practically appreciated when it is realized that the white-light fringes with a 2-volt pocket-lamp source are brighter than the fringes of the standard Rayleigh with a Pointolite lamp source. While this is of value in making the apparatus more portable, the real advantage lies in the fact that it now becomes possible to carry out refractive-index measurements with the lines of helium, neon, etc., while previously the use of the Rayleigh with monochromatic light was confined to the stronger lines of mercury.

§4. THEORY OF THE SEPARATION AND INTENSITY OF THE FRINGES

The separation and intensity-distribution curves of the fringes bear little relation to those of the usual Rayleigh double-slit fringes, so that a theoretical analysis is desirable. It will be found that the fringe-displacement is not strictly proportional to the retardation, although the discrepancy is so small that it need not be taken into account in practical work. Suppose we have a path-difference p between the two beams entering the telescope. The resultant amplitude A_θ of the light diffracted in a direction θ due to two contiguous apertures of width a will be p A_θ θ, a

$$2A_N \frac{\sin(\pi a\theta/\lambda)}{\pi a\theta/\lambda} \cdot \cos\left(\frac{\pi a\theta}{\lambda} + \beta\right) \qquad \text{......(1)},$$

when 2β is the phase-difference, for a wave-length λ, due to the optical path-difference p, and A_N is the amplitude due to one slit in the normal direction. If X be written for $\pi a\theta/\lambda$ the expression becomes β A_N, X

$$2A_N \frac{\sin X}{X} \cdot \cos(X + \beta) \qquad \text{......(1}\,a\text{)}.$$

The distribution of amplitude in the fiduciary half of the field of view is given by the above expression when $\beta = 0$. This is proportional to $\sin\alpha/\alpha$, where $\alpha = 2X$ and corresponds to the pattern of a *single* slit of width $2a$. α

If the central maximum is given for $\theta = 0$, the first minimum (zero) on either side corresponds to $\theta = \pm\lambda/2a$ and the secondary maxima occur at $\pm 1{\cdot}43\lambda/2a$. The separation of the fringes is then $0{\cdot}715\lambda/a$ and not λ/a as is found by considering the variation in the cosine term alone. For convenience, Schwerdt's factor has been determined more closely and it is found to be $1{\cdot}43029$.

For a fixed phase-difference 2β between the beams, the positions of the maxima are given by equating to zero the differential coefficient of the expression (1 a) with respect to X, whereupon we get the following equation:

$$\tan\beta = \frac{2X\cos 2X - \sin 2X}{2X\sin 2X - 2\sin^2 X} \qquad \text{......(2)}.$$

When β (half the phase-retardation) $= 0$ or π; $\tan\beta = 0$ and, from equation (2),

$2X \cos 2X = \sin 2X$. This is of the well-known form $\alpha = \tan \alpha$, giving maxima when $2X = 0$ and $2X = 1{\cdot}43029\pi$. The angle between the fiduciary fringes is then

$$\theta = 1{\cdot}43029\lambda/2a \qquad \ldots\ldots(3)$$

and corresponds also to a path-difference of one wave-length between the beams.

The true path retardation, given by $\lambda\beta\pi^{-1}$, has been calculated from equation (2) for various values of the position of the fringe-maximum expressed as fractions of $\frac{1}{2} \times 1{\cdot}43029\pi$ (i.e. of $128^\circ\ 43'\ 36''$), and the results are given in the table, and in the graph of figure 3.

Table

Observed fractional fringe-displacement	0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1
Actual retardation causing above (wave-lengths)	0	·0954	·1910	·2875	·3938	·4815	·5806	·6813	·7843	·8903	1
Correction	0	·0046	·009	·0125	·0162	·0185	·0194	·0186	·0157	·0097	0

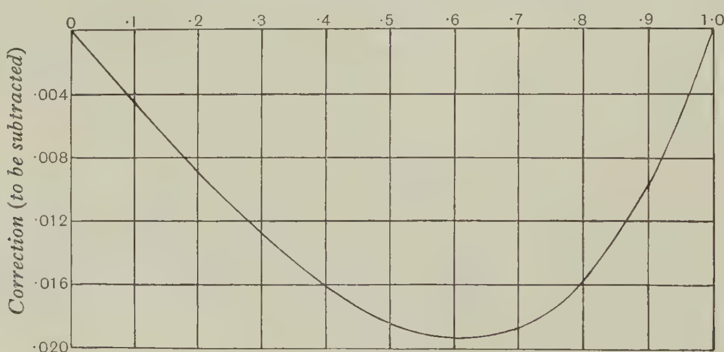

Fig. 3.

The observed fringe-displacement is, in general, slightly greater than the retardation (measured in wave-lengths) producing it, but the effect is so small as to be practically unimportant. The greatest deviation is for a retardation of approximately 0·6 of a fringe, when the discrepancy is less than one-fiftieth of the fringe width. Since these fringes approximate to a $\sin^2 \theta$ intensity curve, they are rather broad and it is hardly likely that a fractional part can be determined to within this accuracy unless very special methods are employed.

It must be noted that this lack of linearity does not in any way affect visual determinations where coincidence is obtained between the two sets of fringes by tilting the compensator plates, since the correction term is zero when the total path-difference is any integral multiple of the wave-length employed. It could only be of importance in cases such as the method outlined in the following section, where the fractional part is determined by actual measurement and not by compensation methods.

§5. PROPOSED NEW METHOD OF DETERMINING THE DISPERSION CURVES OF GASES AND LIQUIDS

The very considerable gain in brightness of the fringes afforded by this method makes it possible to combine the instrument with a spectrograph, and by applying a variation of Benoit's fractional-part method, discussed below, the refractive indices of a substance for a large number of wave-lengths can be simultaneously determined.

For this purpose, the primary collimator slit and the Albrecht rhombs are mounted horizontally. The gas or liquid cells are mounted one above the other in one half of the beam and the compensator system is also rotated through 90° so that the two sets of fringes are now horizontal. These fringes are focussed on the slit of a spectrograph, and the arrangement is shown diagrammatically in figure 4.

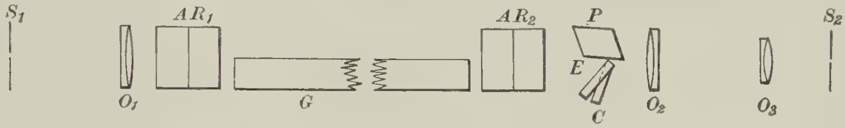

Fig. 4.

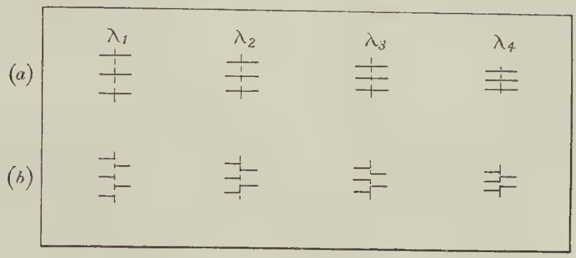

Fig. 5.

In addition to the Albrecht rhombs AR_1 and AR_2, a special astigmatic lens system O_3 is required. This lens, which must also be achromatic, has the plane of the vertical slit S_2 of the spectrograph as its focal plane, so that the horizontal fringes are in sharp focus. Simultaneously it should form, in the centre of S_2, an image of the vertical dividing edge E of the lifting prism P.

The appearance of the photographic plate for a wide slit S_2, with equal retardation in the gas tubes G and the compensator plates C parallel, is shown diagrammatically in figure 5 (*a*), the patterns in the two halves (divided by the broken line) for any wave-length being the same. As the wave-length becomes shorter, the distance between the orders becomes correspondingly shorter. Figure 5 (*b*) shows the appearance of the fringes, with a fixed path-difference between the two beams, due either to a difference of index in the tubes or to a relative tilt between the compensator plates. One half of the pattern remains as in figure 5 (*a*), the other half being displaced as shown. This displacement, measured in terms of the distance between the fringes in the other half, gives the fractional part of the retardation.

The proposed method of obtaining the dispersion curves for a gas is as follows. A photograph is taken, with a suitable source and with the two gas tubes exhausted. This should give a pattern similar to figure 5 (*a*), and affords a check of the correct adjustment of the apparatus. A small amount of gas is allowed to pass into one tube so that the pressure p at temperature t can be determined with fair accuracy. The tap to the tube is then closed and a photograph is taken on the same plate. This pressure is chosen to give an average displacement of one fringe. Suppose
f_1 that for a particular wave-length it is $(1 + f_1)$. A further photograph with a pressure $4p$, say, should give a displacement of $(4 + 4f_1)$ fringes, but the new fraction f_2 may be slightly different. The previous exposure now decides the integral number of wave-lengths n and the correct value for $4p$ will be $(n + f_2)$. A third exposure with a pressure of $16p$ should in turn be $(4n + 4f_2)$ and the exact fraction can be determined from this exposure. This is continued until a sufficient number of fringes have been displaced so that an experimental error of $\pm$ 0·05 or $\pm$ 0·1 of a fringe in the final measurement will still give the required percentage accuracy.

The method has several advantages. The data for a large number of wave-lengths are simultaneously collected on a single plate, so that one naturally exercises greater care in the determination of the final temperature and pressure than if independent determinations for each line have to be carried out. By suitable choice of quartz prisms and the use of quartz-sylvine-fluorspar objectives, the whole range from about 2000 Å. to 8000 Å. or 10,000 Å. can be covered. If the observer is willing to refocus the apparatus for different regions of the spectrum, the much less expensive quartz-fluorspar doublets may be used as objectives.

If at a given temperature the density of the gas does not vary linearly with the pressure, the method outlined above will give spurious results. Its accuracy can be checked by using another ratio of pressure-change with a second plate, and if the final fractional part differs appreciably the inference is that Boyle's law is not obeyed. Such cases can be investigated by the same method, provided the changes of pressure are made so small that no confusion as to the value of the integer of the fringe-displacement exists at any pressure in spite of the non-linearity of the curve. The data thus obtained would enable an accurate curve of the relationship between the density and the pressure for the substance in question to be drawn.

In applying the method to liquids, the simplest way is to have pairs of plates of quartz of equal thickness. Suppose the length of the liquid cells is 1 cm., convenient values of the thickness of the plates would be 0·99, 0·97, 0·91, 0·66 cm. These pairs, being cut from the same original plate, produce no fringe-displacement when placed one in each cell, so that effectively we have liquid cells of length 0·01, 0·03, 0·09, 0·27 and 1 cm. The exact values are unimportant provided they can be determined to within the requisite accuracy. A subsidiary determination with the Abbe or Pulfrich refractometer will enable the integer value for the fringe-displacement for the 0·01 cm. cell to be determined with certainty. The further procedure is the same as for gas determinations, but care must be taken with certain liquids that evaporation does not influence the results.

Messrs Adam Hilger Ltd. have, at the writer's suggestion, produced a cross-

slide photomeasuring micrometer. This instrument was primarily designed to measure the diameters of rings in Fabry-Perot-interferometer and reflection-échelon photographs. It consists of the usual long-screw photomeasuring micrometer L 13 and a short cross slide with a total traverse of only 2 cm. When required, the microscope can be rapidly transferred from one line to another and the arrangement is admirably suited for measuring the fringes described above. The long-screw scale need only be used when the identification of any line is a matter of doubt, as may be the case with, say, the iron spectrum.

§ 6. ACKNOWLEDGMENTS

In conclusion I wish to express my thanks to Mr T. T. Thomas, M.A., who has kindly undertaken the laborious task of calculating the table and prepared the graph of figure 3; and also to the technical staff of Messrs A. Hilger Ltd., who are the sole manufacturers of the instrument, for their cooperation in the experimental work.

DISCUSSION

Mr C. V. Jackson. The author has greatly under-rated the degree of accuracy which can be obtained with the Fabry and Perot interferometer. Thus, he states that it is possible to make measurements to 1 or 2 per cent of a wave in the case of the best fringes. Actually considerable experience, which I have gained in the course of interferometric determinations of wave-length, has shown me that even in the case of very poor fringes an accuracy of about 1 per cent can be reached, while for sharp fringes I have often found that my measurements are right to $\frac{1}{4}$ per cent or even better. In the measurement of the refractive index of air by means of the Fabry and Perot interferometer, the errors introduced by incorrect phase-corrections, or a slight error in the scale of the rings system, is of no consequence. For this reason there should be little difficulty in attaining an accuracy of about $\pm$ 0·0001 Å. by using this method. It is extremely doubtful if an accuracy as high as this could be attained by means of the method suggested by the author.

The author suggests that errors of about $\pm$ 0·0002 Å. may be introduced by the use of Meggers and Peters's tables, but I think this is very improbable on account of the good agreement between observed values in the region 2300–3500 and wave-lengths calculated from lines in the visible. If there were errors of the magnitude suggested by the author they would certainly upset this agreement, since all lines must have their wave-lengths reduced to vacuum for calculating from term values. I can see no advantage in the author's method over that used by Meggers and Peters.

Dr J. J. Fox. The author has done a service in calling attention to the comparative inadequacy of the data for the refractive index of air, when the progress in the accuracy of the determination of wave-lengths of spectral lines is considered. The most extensive determinations of the refractive index of air are due to Meggers

and Peters, carried out in 1918. The data obtained were doubtless sufficient to yield a more accurate dispersion formula than any other available, yet even it does not produce results better than those indicated in the paper. This suggests the desirability of securing some method or instrument capable of giving greater accuracy, especially when one wishes to determine refractive indices of gases both accurately and conveniently. The question of the desirability of reconsidering even the refractive index of air is brought out by the calculations of Opladen*, who derived the value $293{\cdot}03 \pm 0{\cdot}17$ for $(n - 1) \times 10^6$ for λ 5461 from the best values available, while the value calculated from Meggers and Peters's formula is about 5 units lower in the seventh decimal place. The procedure described by the author has been found by us (at the Government Laboratory) to meet the requirements, inasmuch as it gives fringes of much greater brightness, permitting of the use of fainter spectral lines than hitherto and giving more comfort in visual observation.

AUTHOR'S reply. Mr C. V. Jackson appears to have misunderstood my purpose. I tried to show that by employing lateral displacements and subsequent recombinations, the intensity of the fringe system in a divided-wave-front interferometer can be very considerably increased while, as is well known, such displacements of a beam will have no effect on the brightness of an optical image, as distinct from an interference fringe. Further, a type of interference system is described in which the fringe-displacement is not linearly proportional to the retardation or path-difference interposed. This, as far as I have been able to find, is the first instance in interferometry where such non-linearity has been found to exist.

The section on the refractive index of air, to which Mr Jackson takes such strong exception, is not strictly germane to the main substance of the paper, but I could not resist the opportunity of discussing this very important point for the reasons given below.

Apart from special purposes such as those of metrology, the real purpose of accurate wave-length determinations is to obtain the frequencies of these radiations, and the results so obtained can be used both to evaluate certain fundamental constants as well as to test the validity of theories of spectral emission. The primary standard is the wave-length of red cadmium radiation emitted under specified conditions and measured in dry air, the temperature and pressure of which are specified. The value adopted is a weighted mean of the independent results of Michelson and of Buisson, Fabry and Perot. The choice of secondary and tertiary standards has been made with great caution by the International Astronomical Union; values have not been accepted unless there was a sufficiently good agreement between the determinations made by independent observers.

Yet the real significance of all this, and of a great deal more work for which these standards have been used, is made to depend, in turn, on a single set of measurements of the refractive index of air which were carried out over fourteen

* *Z. Physik*, **42**, 160 (1927).

years ago, since which time there has been a considerable improvement in both the technique and the apparatus for precision wave-length measurements.

My estimate of the probable accuracy of Meggers and Peters's results (3 or 4 units in the seventh decimal place) is the value the authors quote as the probable error of a single determination. They do not specifically give an estimate of the probable accuracy of a value derived from their equations, but it can be inferred from the temperature and pressure tolerances mentioned that a probable accuracy of one unit should be attained. I feel that the considerable number of the readings taken does not so much appreciably increase the final accuracy of the value for any given wave-length, as determine in better detail the general shape of the dispersion curve.

I cannot accept Mr Jackson's assumption that the experimental agreement of tests of the combination principle is necessarily a proof of the accuracy of Meggers and Peters's results. Let us take an extreme case by way of illustration; suppose a line A at 10,000 Å. should be given by the difference between lines (C) at 2000 Å. and (B) at 2500 Å. The wave-number corrections for these values are usually taken as $A = -2{\cdot}738$, $B = -12{\cdot}053$ and $C = -16{\cdot}274$. If we assume that Meggers and Peters's results are uniformly 6 units in the seventh place too low, the following values: $A_1, -{\cdot}006$; $B_1, -{\cdot}024$; $C_1, -{\cdot}030$, have to be added to the above corrections. It will be noticed that $C_1 - B_1 = A_1$, so that the same agreement will be obtained in both cases.

Meggers and Peters's formula for the index of dry air under standard conditions gives for the red cadmium wave-length

$$(\mu_{15} - 1) \times 10^7 = 2758{\cdot}14.$$

The determination by Perard* for this wave-length, carried out at the Bureau Internationale des Poids et Mesures, gives the value 2764·13. I understand that as a result of a careful determination with étalons of far greater plate-separations than those used by Meggers and Peters, the Metrological Department of the National Physical Laboratory has arrived at a provisional value which is in very close agreement with the result of Perard.

This difference of six units in the seventh place introduces an uncertainty of $\pm {\cdot}0038$ Å. in the vacuum value of this wave-length. In brief, until this question is finally settled, wave-length determinations in air will have little real significance beyond the second decimal place in Ångstroms.

To return to Mr Jackson's comments, he estimates that there should be little difficulty in attaining an accuracy of $\pm$ 0·0001 Å. in the vacuum correction with the Fabry-Perot method. For the wave-length we are considering, this would require an accuracy of $\pm$ 1·5 units in the eighth decimal place for the refractive index, or that $(\mu - 1)$ must be obtained to within one part in 20,000. Taking the value of $\frac{1}{4}$ per cent as the possible error in the determination of the fractional part, which he quotes as practicable in the case of sharp fringes, the change in the order of

* A. Perard, *Phys. et Ra.* **6**, 217 (1925); see also *Trav. et Mem. Bur. Internat.* **18**, 34 (1929).

interference must be about 100 fringes to secure that the maximum possible error in the two fractional-part determinations (0·005) shall be in the correct proportion.

Since $(\mu - 1) = (n_{\text{air}} - n_{\text{vac.}})/n_{\text{vac.}} = 2764 \times 10^{-7}$, and $n_{\text{vac.}} \doteqdot 360{,}000$,

and the separation of the interferometer plates would have to be approximately 12 cm. With this separation, the fringes from even the red cadmium line will be very poor, so that the $\frac{1}{4}$ per cent accuracy in fractional-part determination would be utterly impossible. The 1 per cent accuracy suggested for poor fringes would require a plate-separation of 48 cm., for which distance no sign of a fringe would be seen. It is thus obvious that there is no hope of attaining this order of accuracy by means of the differential method employed.

I think the other opinions expressed by Mr Jackson are sufficiently answered in Dr Fox's communication. I particularly appreciate Dr Fox's comments, as he and his colleagues at the Government Laboratory are the only other investigators who have, as yet, had an opportunity of actually using the new instrument.

Postscript added during proof. Through the courtesy of Dr Lampe I have been able to examine the apparatus used and the data obtained for the refractive index of air, at the Physikalische Technische Reichanstalt. They employed a path-length of 4 metres, and their value for the index for this wave-length is again in very close agreement with that of Perard. This establishes beyond all doubt that the tables are in error and should be speedily revised.

ON THE REPRESENTATION AND CALCULATION OF THE RESULTS OF GRAVITY SURVEYS WITH TORSION BALANCES

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ABSTRACT. This paper deals with an alternative and, it is believed, superior method of indicating the functions of the Eötvös torsion balance, and of calculating those quantities, depending upon the distortion of the earth's gravitational field, which the balance measures. Remarks are also made on the representation of these quantities—the so-called *gravity-gradient* and *horizontal directive tendency*—and a convenient method is indicated of applying graphically the necessary corrections for the effects of earth's rotation and irregularity of the surface of the ground.

In most, if not all, of the many surveys of the departures from uniformity, or, more strictly, spherical symmetry, of the earth's gravitational field, carried out with torsion balances of the Eötvös type, it has been the practice to proceed directly to the calculation of the coefficients in the fundamental equation originally shown by Eötvös to be applicable to such instruments. Thereafter these coefficients have been used to derive certain other related quantities which are, in fact, those usually represented on the survey maps to show the state of the gravitational field. It is the purpose of this paper to propose a modification of the form of the fundamental equation such that the equation may contain explicitly from the outset the actual quantities which have ultimately to be calculated and used for representation. The result is that not only is the evaluation rendered more direct, but the significance of the terms in the equation, in relation to the measuring functions of the instrument, becomes much more obvious.

The Eötvös form of the fundamental equation* is

$$C_\alpha = \frac{K_1}{2}\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right)\sin 2\alpha + K_1 \frac{\partial^2 U}{\partial x\,\partial y}\cos 2\alpha + K_2\left(\frac{\partial^2 U}{\partial y\,\partial z}\cos\alpha - \frac{\partial^2 U}{\partial x\,\partial z}\sin\alpha\right) \quad \ldots\ldots(1).$$

Here C_α is the resultant horizontal force-moment acting on the suspended beam C_α
system, due to the non-uniformity (supposed small) of the gravitational field. This moment varies with the angle α which the vertical plane of symmetry of the beam α
system makes with the arbitrarily chosen vertical plane x, z. The constants K_1 and K_1

* *Ann. der Phys.* **59**, 364 (1896). The derivation of the equation will also be found in the paper by Shaw and Lancaster-Jones, *Proc. Phys. Soc.* **35**, 152 (1923).

K_2

K_2 are instrumental. In the original Eötvös instrument K_1 was very nearly the moment of inertia of the whole beam system (which was confined nearly to one plane) about the axis of suspension; and K_2 differed very little from mhl, where

m, h, l

m is the mass of the lower weight, h its mean distance below the beam, and l its mean distance from the axis of suspension. In later forms of the beam it has been necessary to calculate K_1 and K_2 for the more complicated mass distribution by means of the general formulae

$$K_1 = \int(\xi^2 - \eta^2)\, dm \quad \text{and} \quad K_2 = \int \xi z\, dm,$$

where ξ, η and z are the rectangular coordinates of the element of mass dm, the origin being on the axis of suspension, while ξ and η are horizontal and respectively in and perpendicular to the symmetrical vertical plane of the beam, and z is vertically downwards. It may be noted that in the Shaw and Lancaster-Jones gradiometer the mass-distribution of the beam is so chosen as to make K_1 equal to 0, and that, in the Coulomb form of balance, symmetry with regard to a horizontal plane makes K_2 vanish.

U

The partial differential coefficients in the fundamental equation refer to the gravitational potential U. Frequently they are represented, for brevity, by the notation

U_Δ, U_{xy}

U_{xz}, U_{yz}

$$U_\Delta = \frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}, \quad U_{xy} = \frac{\partial^2 U}{\partial x\, \partial y},$$

$$U_{xz} = \frac{\partial^2 U}{\partial x\, \partial z} \quad \text{and} \quad U_{yz} = \frac{\partial^2 U}{\partial y\, \partial z},$$

and these four coefficients constitute the unknown quantities ordinarily derived in the first instance from the observations at any particular station, in a manner which will be described later.

These coefficients, although not giving a complete description of the field-variation (since the coefficient $\partial^2 U/\partial z^2$ is not determined by the apparatus), are related closely to two more fundamental quantities which have both magnitude and direction. Eötvös gave them the names *die horizontale Richtkraft* and *der Gradient der Schwerkraft*, and they are commonly known in English as the *horizontal*

H.D.T.

directive tendency (or H.D.T.) and the *gravity-gradient* respectively. Although these names are open to criticism, it remains true that the quantities in question are of such special significance in describing the state of the gravitational field that they are in most cases chosen for representation upon the survey maps.

g, s

The gravity-gradient, strictly the maximum gradient in a horizontal direction of the vertical gravitational intensity, or $\partial g/\partial s$, where s is the direction of maximum variation, is a vector. It can, if desired, be regarded as having, in any two horizontal directions x and y, two rectangular components $\partial g/\partial x$ and $\partial g/\partial y$, fulfilling the relations

$$\frac{\partial g}{\partial x} = \frac{\partial g}{\partial s} \cos \phi, \quad \frac{\partial g}{\partial y} = \frac{\partial g}{\partial s} \sin \phi,$$

ϕ

and their implications, ϕ being the angle between s and x. Since $g = \partial U/\partial z$,

$\partial^2 U/\partial x\,\partial z$ and $\partial^2 U/\partial y\,\partial z$ are identified with $\partial g/\partial x$ and $\partial g/\partial y$ respectively; so that, if we denote the maximum gravity-gradient, $\partial g/\partial s$, by G, we have G

$$U_{xz} = \frac{\partial^2 U}{\partial x\,\partial z} = G \cos\phi \qquad \text{......(2)}$$

and

$$U_{yz} = \frac{\partial^2 U}{\partial y\,\partial z} = G \sin\phi \qquad \text{......(3).}$$

A knowledge of U_{xz} and U_{yz} thus implies a knowledge of G and ϕ, and conversely.

This deals with the transformation of two of the unknowns in the fundamental Eötvös equation. The remaining two, namely, U_Δ and U_{xy}, are related to the horizontal directive tendency, but in a rather different way. This name originated from a consideration of the behaviour of a balance beam of the Coulomb type, for example, a light horizontal rod with equal masses at the ends, in relation to the curvatures of the equipotential or "level" gravitational surface. Such a beam would tend to set itself so that its axis lay in the vertical plane corresponding to the least curvature of the level surface, curvature being reckoned positive downwards. The beam would also be in equilibrium, though now unstable, if lying in the plane of greatest curvature of the local equipotential surface. A maximum value of the controlling couple is reached when the axis of the beam bisects the angle between these two orthogonal planes. The value of this maximum couple is proportional to $g\,(c_1 - c_2)$, where c_1 and c_2 are the maximum and minimum curvatures of the level surface at the point of observation, and this product of the gravitational intensity and the curvature-difference is, in fact, identified with the so-called horizontal directive tendency. It is really a property of the field, not dependent on reference to any instrument, at any rate in a formal manner. For this reason the name in English is not a good one; but a more serious objection to it is that the gravity-gradient can equally be regarded as a horizontal directive tendency in respect of an apparatus having the full properties of the Eötvös instrument. For in that case the beam tends also to rotate horizontally into the direction of maximum gradient. The name in German does not suffer from this defect, but the quantity in question is not a force, as this name suggests. Actually the gravity-gradient and the horizontal directive tendency both have the same dimensions, namely, $1/\text{sec}^2$. It is perhaps too late in the day to suggest a new name, and the name "horizontal directive tendency" will therefore be used, somewhat reluctantly, in the rest of this paper. Several other names have been proposed, such as "curvature-value" or "curvature-difference," but they lack the desirable precision*. c_1, c_2

The quantity itself has magnitude and direction, but it possesses the latter in a limited sense. It is not a true vector, for its direction has no "sense." It is unaltered by rotation through π, but turning through $\frac{1}{2}\pi$ changes its sign. Its

* I have discussed this question with some mathematical colleagues. One of them has made the rather attractive suggestion that the difficulty might be solved by the invention of a composite word. He proposed *graviture* for the combination of gravity and curvature implied by a product of the type gc; then *graviture-difference*, or, more strictly, *principal graviture-difference*, would automatically signify the function $g\,(c_1 - c_2)$.

direction is taken to be that corresponding to stability, namely, the horizontal direction in which the vertical downward curvature of the equipotential surface
R is least. If we denote the scalar value of $g(c_1 - c_2)$ by R, and its direction, with
θ respect to the x axis, by θ, it is related to U_Δ and U_{xy} by the equations

$$2U_{xy} = 2\frac{\partial^2 U}{\partial x \partial y} = R \sin 2\theta \qquad \text{......(4)},$$

$$-U_\Delta = \frac{\partial^2 U}{\partial x^2} - \frac{\partial^2 U}{\partial y^2} = R \cos 2\theta \qquad \text{......(5)}.$$

These transformations complete what is required for our purpose.

Returning to the fundamental equation (1), we re-write it

$$C_\alpha = \frac{K_1}{2}\left[2\frac{\partial^2 U}{\partial x \partial y}\cos 2\alpha - \left(\frac{\partial^2 U}{\partial x^2} - \frac{\partial^2 U}{\partial y^2}\right)\sin 2\alpha\right]$$
$$+ K_2\left[\frac{\partial^2 U}{\partial y \partial z}\cos\alpha - \frac{\partial^2 U}{\partial x \partial z}\sin\alpha\right],$$

and, by substitution from (2), (3), (4) and (5), we obtain

$$C_\alpha = \tfrac{1}{2}K_1 R(\sin 2\theta \cos 2\alpha - \cos 2\theta \sin 2\alpha)$$
$$+ K_2 G(\sin\phi\cos\alpha - \cos\phi\sin\alpha),$$

or,

$$C_\alpha = \tfrac{1}{2}K_1 R \sin 2(\theta - \alpha) + K_2 G \sin(\phi - \alpha) \qquad \text{......(6)}.$$

This is the form of the equation which is suggested for use in preference to that containing the U coefficients. It contains another, but related, set of four unknown quantities, R, θ and G, ϕ, which equally describe the field, and are already in the form required for map-making purposes. At the same time the new equation makes more evident the manner in which the total torsional control is made up of two distinct contributions, namely, that due to the horizontal directive tendency and that due to the gravity-gradient. If, for example, the azimuth, α, of the beam system happens to coincide with that of G, namely, ϕ, or is in the exactly opposite direction, the torsional control, C_α, is wholly due to R, i.e. to the difference between the principal curvatures of the level surface, in conjunction with gravity. If, on the other hand, the beam azimuth is inclined at any integral multiple of $\frac{1}{2}\pi$ to θ, the direction of R, the gravity-gradient G becomes wholly responsible for the torsion. The magnitudes of the two effects can accordingly be considered separately.

The effect of G resembles that of a uniform force-field, in that the force-moment is proportional to the sine of the angle between it and the beam-axis; and the instrumental constant—the "moment" of the beam from the present point of view—is K_2, which depends on the asymmetrical vertical distribution of mass, in the manner already indicated. Similarly, the other instrumental constant, $\frac{1}{2}K_1$, is a different "moment" which responds to the particular non-uniformity of field, represented by R; only in this case the rule for the calculation of the torsion is different, since the latter is now proportional to the sine of *twice* the angle between the beam-axis and R.

The new form of the equation also makes more evident the distinctive features

of certain particular designs of the torsion balance. If, for example, the moment K_2 is deliberately made negligible, as it would be for a Coulomb balance beam, we see that the second term of equation (6) vanishes, and the instrument becomes irresponsive to gravity-gradient. The equation for such beams reduces to

$$C_\alpha = \tfrac{1}{2}K_1 R \sin 2(\theta - \alpha) \qquad \text{......(7)},$$

which forms the basis for the calculation of the magnitude and direction of the horizontal directive tendency only. Alternatively, should it be desired to measure the gravity-gradient vector only, the mass-distribution of the beam may be designed so as to render that other moment, $\tfrac{1}{2}K_1$, sufficiently small, and equation (6) becomes

$$C_\alpha = K_2 G \sin(\phi - \alpha) \qquad \text{......(8)},$$

which is applicable to this special form of instrument. This is the principle applied in that elegant instrument, the gravity-gradiometer* of Shaw and Lancaster-Jones, which measures only the magnitude and direction of the gravity-gradient, giving negligible response to the horizontal directive tendency. It is evidently possible also to arrange the mass-distribution and degree of symmetry of the beam so as to fulfil the two conditions $K_1 = \int(\xi^2 - \eta^2)\, dm = 0$ and $K_2 = \int \xi z\, dm = 0$; but this would serve no useful purpose, as the instrument would then measure no feature of the non-uniformity of the gravitational field.

The method of derivation of the four unknowns R, θ, G and ϕ (instead of U_Δ, U_{xy}, U_{xz} and U_{yz}) differs somewhat from the normal practice, but it leads to no less simplicity in the calculations from the observations. If we introduce into equation (6) the customary substitutions for C_α, we obtain

$$n_\alpha - n = \tau^{-1} D K_1 R \sin 2(\theta - \alpha) + \tau^{-1} 2 D K_2 G \sin(\phi - \alpha) \quad \text{......(9)},$$

where n_α is the scale-reading when the beam has the azimuth α, and n is the (unknown) scale-reading corresponding to complete absence of torsion in the suspending wire. D is the ratio of the distance between the mirror and the scale to the width of one scale division, and τ is the torsion couple per unit twist of the suspending wire. It should be noted that in practice the torsion angle of the suspension is always very small, being seldom as much as one degree, so that the azimuth of the beam is practically the same as the rotation of the whole instrument from the standard direction. Writing $\tau^{-1}DK_1 = a$ and $2\tau^{-1}DK_2 = b$, both these being purely instrumental constants previously determinable, we obtain from equation (9)

n_α, n

D

τ

a, b

$$n_\alpha - n = aR \sin 2(\theta - \alpha) + bG \sin(\phi - \alpha) \qquad \text{......(10)}.$$

Since n is unknown, as well as R, θ, G and ϕ, at least five equations are required, and these are obtained by taking observations (n_α) in a sufficient number of different azimuths (α). As in the employment of the usual form of the equation, the solution is simplified by using six† azimuths, equally spaced in a complete rotation of the instrument, the supernumerary equation in addition providing a check on the consistency of the observations. The following six equations are thus developed

* The *Mining Magazine*, May, 1929, contains a brief description of this instrument.

† *Note:* Actual settings are commonly reduced to three by working with double-beam instruments.

from (10), corresponding to the values 0°, 60°, 120°, 180°, 240°, and 300° respectively:

n_0 $$n_0 - n = aR \sin 2\theta \qquad\qquad + bG \sin \phi,$$
n_1 $$n_1 - n = aR \sin (2\theta - \tfrac{2}{3}\pi) + bG \sin (\phi - \tfrac{1}{3}\pi),$$
n_2 $$n_2 - n = aR \sin (2\theta - \tfrac{4}{3}\pi) + bG \sin (\phi - \tfrac{2}{3}\pi),$$
n_3 $$n_3 - n = aR \sin (2\theta - \tfrac{6}{3}\pi) + bG \sin (\phi - \tfrac{3}{3}\pi),$$
n_4 $$n_4 - n = aR \sin (2\theta - \tfrac{8}{3}\pi) + bG \sin (\phi - \tfrac{4}{3}\pi),$$
n_5 $$n_5 - n = aR \sin (2\theta - \tfrac{10}{3}\pi) + bG \sin (\phi - \tfrac{5}{3}\pi).$$

In the above the suffix of n indicates the multiple of 60° which is the corresponding azimuth; thus, n_4 corresponds to $4 \times 60° = 240°$.

It is unnecessary here to go through the steps of the evaluations of the five unknowns; it will suffice to indicate that by appropriate selection and addition or subtraction of equations, and by the use of well-known trigonometrical relations, the following results are obtained:

$$6n = n_0 + n_1 + n_2 + n_3 + n_4 + n_5{}^* \qquad \text{......(11a)},$$

which gives the natural zero of the instrument;

$$3n = n_0 + n_2 + n_4 = n_1 + n_3 + n_5 \qquad \text{......(11b)},$$

which provides the check above mentioned;

$$\cot 2\theta = (n_2 + n_5 - n_1 - n_4)/\sqrt{3} \, . \, (n_0 + n_3 - 2n) \qquad \text{......(11c)},$$
$$R = (n_0 + n_3 - 2n)/2a \sin 2\theta \qquad \text{......(11d)};$$
$$\cot \phi = (n_4 + n_5 - n_1 - n_2)/\sqrt{3} \, . \, (n_0 - n_3) \qquad \text{......(11e)},$$
$$G = (n_0 - n_3)/2b \sin \phi \qquad \text{......(11f)}.$$

The two possible values of each of the angles 2θ and ϕ are found from (11c) and (11e), by reference to tables; and application in (11d) and (11f), respectively, enables the magnitudes of R and G to be calculated simply, besides removing the ambiguity of 2θ and ϕ. Thus R and G are determined both in magnitude and direction, and may be represented graphically on the survey map, as is customary, by straight lines in the appropriate directions and of lengths proportional to the magnitudes on a suitably chosen scale.

It may be noted that, for similar purposes, the usual method of evaluating first the coefficients U_Δ, U_{xy}, U_{xz} and U_{yz} requires the additional calculations corresponding to the relations

$$R = \sqrt{(U_\Delta{}^2 + 4U_{xy}{}^2)}, \qquad \tan 2\theta = -2U_{xy}/U_\Delta,$$
$$G = \sqrt{(U_{xz}{}^2 + U_{yz}{}^2)} \text{ and } \tan \phi = U_{yz}/U_{xz}.$$

The question may naturally be raised as to the advantage of obtaining the instrumental measures of R, θ, G and ϕ in the direct manner indicated, when, in fact, they are *resultant* effects attributable only partly to the actual gravitational

* This is only a special case of a general rule. If any number ν (necessarily integral) of azimuths are spaced with equal intervals in a complete rotation, it can be shown that

$$\nu n = n_0 + n_1 + n_2 + \ldots + n_{\nu-1}.$$

disturbance sought, namely, that due to buried structure. They contain components also for which the rotation of the earth and local topographical features are responsible. It might with some reason be argued that, as the elimination of the latter effects is necessary, it is better to have the resultant horizontal directive tendency and gravity-gradient in the form of x and y components, so as to reduce to scalar subtraction the corrections in question. The answer is that the elimination can be performed still more simply by graphical methods, ordinary vector diagrams being used for the gravity-gradient, and a modified construction for the horizontal directive tendency.

For example, if we denote by G_E the gravity-gradient due to the earth's rotation, its value is given very nearly by G_E

$$G_E = 8 \sin 2\lambda \times 10^{-9} \text{ sec.}^{-2}*$$
$$= 8E \sin 2\lambda,$$ E

where λ is the latitude of the observation station. The direction of the vector G_E is in the northern hemisphere, towards the geographic north. It can be subtracted λ

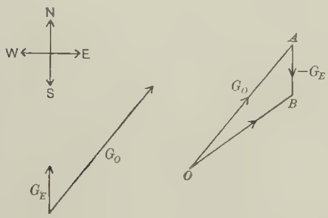

Fig. 1. Graphical method of obtaining difference between gravity-gradients.

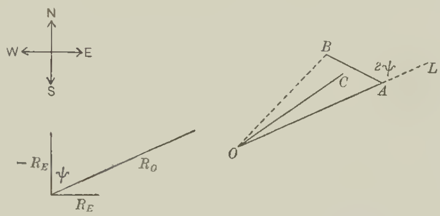

Fig. 2. Graphical method of obtaining difference between horizontal directive tendencies. The resultant, OC, bisects the angle AOB.

from the observed vector G_O by reversal and vector addition as indicated in figure 1. Thus, if G_O and G_E are as shown, the vector diagram consists of OA, representing G_O, and AB, representing G_E reversed; and the remaining side of the triangle, OB, represents in magnitude and direction the gravity-gradient after the removal of the earth's rotation effect. Clearly other corrections may be made by repeated application of like rules, provided the correction data are available as vectors; and the final result is obtained from a single diagram in the form in which it is required, namely, a straight line of particular length and direction, representing completely the contribution of the underground structure to the local gravity-gradient. G_O

The construction for the horizontal directive tendency is different, owing to the peculiar properties of this doubly-directed quantity. To subtract one from another one must first rotate it, not through 180°, but through 90°, and then perform the addition by a special set of rules. The angle between the two directions has to be doubled in the equivalent of a vector diagram, and the completing side

* 1 Eötvös unit, denoted by E., is equal to one thousand-millionth of a c.g.s. unit, or 10^{-9} sec.$^{-2}$, and is the small unit usually employed in torsion-balance work.

then represents the magnitude, but not the direction, of the resultant. The *direction* of the resultant *bisects* the internal angle between the initial and final sides. Thus,
R_O in order to eliminate from the observed horizontal directive tendency, R_O, the part
R_E R_E, due to the earth's rotation, given nearly by

$$R_E = 10E \cos^2 \lambda,$$

and directed east and west geographically, we represent R_O by OA, figure 2. We add $-R_E$, which has a north-south direction as already explained, by constructing
ψ the line AB so that the angle LAB equals twice the angle ψ between $-R_E$ and R_O, and making AB proportional to $-R_E$ on the scale adopted. Then, on the same scale, OB represents the magnitude of the resultant horizontal directive tendency, and its direction is along OC, where OC bisects the angle AOB. Thus, if OC is also made equal to OB, it represents in all respects the horizontal-directive-tendency element of gravitational distortion, other than that for which the earth's rotation is responsible. Here again other corrections, notably the topographical one, can be applied by continuing the use of similar rules, which now give a polygon instead of a triangle, to obtain finally the graphical representation of the magnitude and direction of the desired horizontal directive tendency.

In this method, then, the whole process of the derivation of the composite (R, θ) and (G, ϕ) from the instrumental observations consists in the construction of two diagrams, using the correction data for the earth's rotation and the various features of ground surface irregularity. The fact that these data, in respect of terrain and topography, are by the usual methods of procedure made available in the first instance as components (of the U_Δ, U_{xy}, U_{xz} and U_{yz} type) presents no difficulty. For in the gradient diagram these reversed components, $-U_{xz}$ and $-U_{yz}$, will simply constitute two successive steps at right angles to one another, instead of the one step which would obtain were their resultant already known. And the reversed horizontal-directive-tendency components, U_Δ and $-2U_{xy}$, will similarly form part of the horizontal-directive-tendency diagram, in place of the single line to which they are equivalent.

It seems worth while to mention a possible extension of the use of these diagrams, namely, to perform part of the integration necessarily involved in estimating the topographical effect. Starting from the station point as centre, each sector of the ground surface, lying between two close azimuths, can be regarded as contributing an element of gravity-gradient, $d(G)$, and an element of horizontal directive tendency, $d(R)$, each, if positive, in the direction of the mean azimuth of the narrow sector. If $d(G)$ is negative, it is directed oppositely to the sector azimuth; if $d(R)$ is negative, it is at right angles to the sector azimuth. The numerical values of the directed quantities $d(G)$ and $d(R)$ can be conveniently estimated by planimeter integration based upon "levelling" observations at selected radial distances, and the final integration for the whole range 2π of azimuths, can then be performed by the diagrammatic rules already described. The method has been worked out in detail, and has several interesting features, one of which is that the diagrams may consist partly of continuous curves such as arcs of circles. Drawings are

substituted largely for numerical calculations, and we still require only two diagrams altogether.

One hesitates, however, to propose definitely at present this development as an alternative superior to the elegant methods, particularly Schweydar's, which have been used widely and successfully in the field for estimating the topographical corrections. The basis of Schweydar's method* necessitates initial resolution into rectangular components, and integration with respect to azimuth *before* that with respect to radial distance. And his convincing use of the Fourier expansion rather suggests that his method deals as precisely as possible with the interpolations among the necessarily limited number of levelling observations. The only really satisfactory comparison of the two methods would be by results; and when sufficient field-practice has been obtained with the graphical method indicated, a fuller account of it may be offered for publication.

* W. Schweydar, "*Die topographische Korrection bei Schweremessungen mittels einer Torsionwage*," *Z. f. Geophys.* **3**, 17 (1927).

DISCUSSION

For discussion see p. 489.

SOME OBSERVATIONS WITH A GRAVITY-GRADIOMETER

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ABSTRACT. This paper gives an account of a series of observations with a Shaw and Lancaster-Jones gravity-gradiometer, during which it was discovered that very persistent, although small, electric charges could be developed on the mica ring forming part of the oscillation-damping system. The origin of these charges has not yet been ascertained precisely, but one of them has been located and measured approximately by suitable manipulation of the instrument. It is shown that such charges, which may persist for weeks, may not arouse suspicion, although in fact they lead to spurious results in the normal use of the instrument. Thus the apparent gravity-gradient determined by an instrument insidiously charged may have errors of magnitude much larger than the true gradient, and errors of direction of many degrees. It is shown that the effect can be effectively eliminated by introducing into the instrument a sufficient quantity of a suitable ionizing agent, such as meso-thorium. This precaution, which can be adopted readily in both old and new gradiometers, is proved to result in a notable improvement in the accuracy and reliability of the instrument.

§ 1. INTRODUCTION

THE observations to be described were made with one of the two gravity-gradiometers used in the work of the Imperial Geophysical Experimental Survey which operated in Australia from 1928–30. It has since been acquired for the newly instituted courses of instruction in applied geophysics in the Imperial College, and it was during the preparation of the instrument, which arrived from Australia in a somewhat damaged condition, that the at first mysterious behaviour was noticed, and the explanation tracked down. When the instrument was purchased for the College it was not known that such great difficulties had been experienced with it in the Australian Survey that it had provided practically no satisfactory results. But evidence is found in the report* that makes it clear that the gradiometer now under consideration is in fact what is there called the "new gradiometer," i.e. the one added to the equipment during the latter part of the Survey; and it is quite apparent, from the remarks on page 321 of the report concerning the instrument, that its erratic behaviour then was due to the same cause which has now been discovered at the Imperial College, and not to imperfections of the torsion wires, as suggested in the report. The origin of the erratic behaviour is persistent electrification of the mica ring incorporated in the instrument for damping purposes, and the effect can be removed by ionizing the air within the instrument. As a result

* *The Principles and Practice of Geophysical Prospecting, being the Report of the Imperial Geophysical Experimental Survey.* (Cambridge University Press, 1931.)

this gradiometer has become docile and reliable instead of petulant and wayward, and now performs its function of measuring the magnitude and direction of the gradient of gravity in a most satisfying manner.

It is just possible, of course, that this particular instrument is peculiar in its liability to electrification; but it would not be safe to assume this, for mica is both an exceptional insulator and easily electrified. As will be seen, the effect can be readily eliminated and the performance and reliability of these elegant instruments thereby greatly improved. It is for this reason mainly, but also as showing the interesting manner in which the disturbing effect was brought to light, that an account of the observations is now presented.

§2. THE GRAVITY-GRADIOMETER

There has appeared as yet* no adequate description of this notable instrument of precision—the gravity-gradiometer. It was invented and designed by Dr H. Shaw and Mr Lancaster-Jones, and the first examples of it, including the one with which the following experiments were made, were manufactured by Messrs L. Oertling, Ltd. The inventors have already given a short account of it, but without instrumental details†. It is accordingly necessary to describe here those features which are related to the subject-matter of this paper. The instrument is a modification of the famous torsion balance of Eötvös‡, which measures small departures from uniformity in a gravitational field. In the Eötvös form the balance consists of a horizontal beam suspended at its mid-point by a suitable torsion wire and carrying at different levels at its ends relatively heavy masses, one being on the level of the beam itself and the other suspended from it by a wire. Such an arrangement can be made with an extraordinarily high sensitivity, and, with suitable precautions, and with observations of the state of twist of the torsion wire in a sufficient number of azimuths of the instrument as a whole, will measure with great precision the magnitude and direction of two quantities which describe partially the non-uniformity of the gravitational field at the place where the instrument is located. The first of these, commonly called the *horizontal directive tendency*, is in fact the difference between the principal curvatures of the local "level" surface multiplied by g, the vertical component of gravity. The second is the so-called *gravity-gradient*, i.e. $\partial g/\partial s$, where s is the horizontal direction of maximum change of g.

The modification due to Shaw and Lancaster-Jones consists in rendering the instrument irresponsive to the former of these two controls, for reasons given by those authors§; so that only the horizontal gravity-gradient is measured. This is secured by a suitable design of the distribution of the masses attached to the beam, and the method chosen has been to construct the beam in the form of a horizontal circular ring of aluminium with its centre directly below the point of suspension, and carrying three gold masses equidistant from the centre in azimuths at intervals

* It is understood that a full account of the instrument will be published shortly in the *Journal of Scientific Instruments*.

† *The Mining Magazine*, **40**, 272 (1929).

‡ *Ann. der Phys.* **59**, 370 (1896).

§ *Loc. cit.*

of 120°. Two of these are nearly in the plane of the ring and the other is supported above, at a height of about 50 cm., by a light aluminium tube. The arrangement is shown in figure 1, which represents also the ring of mica—the main feature for our consideration. This ring, of internal and external diameters about 4 in. and 6 in. respectively, is attached below the aluminium ring, and serves, by its movement when suspended in close proximity over a horizontal brass plate, to damp the oscillations of the beam about the vertical axes. The degree of damping—approximate aperiodicity is usually employed—is very sensitive to the interval between the mica ring and the brass plate below it, and provision has accordingly been made

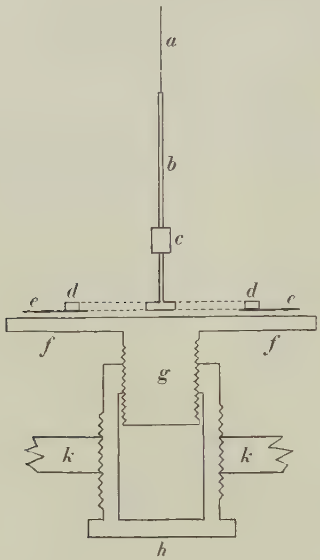

Fig. 1. Diagrammatic section of gradiometer beam system and control of damping plate.

a, lower part of suspending wire.
b, central rod of beam system.
c, mirror frame.
d, aluminium ring carrying gold weights (not in plane of figure and not shown).
e, mica ring.
f, brass damping plate.
g, upper element of differential screw for altering height of *f*, which is prevented from rotating by guides not shown.
h, graduated head of lower element of differential screw, engaging in fixed base of enclosure.
k, base of enclosure containing beam system. The enclosure is fixed to the upper part of a turntable (not shown) on which the whole instrument is mounted, and may be rotated bodily.

for altering very gradually the level of the plate. This is done by means of a differential screw operated by a screw-head outside the enclosure containing the suspended beam. One complete revolution of this screw head raises the plate 0·025 in. and, nominally, a manipulation of $\frac{1}{500}$ of this movement is possible. In practice it is found that approximate aperiodicity is secured by the air damping with about 0·025 in. between the mica ring and the plate, and, under these con-

ditions, the beam reaches equilibrium with sufficient accuracy in from 20 to 25 minutes after having the maximum amplitude of oscillation permitted in the apparatus. This limitation of amplitude is secured by means of adjustable stops, and it is usual in this way to allow an angular range of movement of only about 1 degree.

For purposes of transportation from one observing station to another the delicately suspended beam has to be clamped. This is done by the continued elevation of the damping plate, by means of the differential screw already mentioned, until the mica ring rests upon the plate and is eventually pinched between it and a brass ring fixed above. It should be noted, in relation to the sequel, that this necessary practical manipulation involves frequent contact, not free from friction and pressure, between the mica and the enclosing brass, during both the clamping and the releasing of the suspension of the beam.

It is unnecessary here to describe the means adopted for enclosing the suspended beam so as to prevent effectively the disturbances which might arise from air currents, or the optical system used to observe the angular position of the beam. It will suffice to mention that the divisions of the observation scale are such that about 40 of them correspond to 1 degree of rotation of the beam with respect to the torsion head of the suspension, and that readings can be made accurate to 0·1 division. An essential preliminary adjustment consists of orienting the torsion head carrying the upper end of the torsion wire so that the equilibrium reading, with the instrument located in a gravitational field nearly enough uniform, is approximately midway between those corresponding to the limiting stops. The weight of the beam complete is rather more than 80 gm., and the torsion wires are in the form of ribbons of a platinum alloy (similar to the well-known phosphor-bronze strip), having torsion coefficients (τ) round about 0·1 dyne-cm./radian. τ
Two have been used in the present work—the one originally in the apparatus on arrival from Australia, with $\tau = 0{\cdot}159$ dyne-cm./radian, and a new one supplied by Dr Shaw, with $\tau = 0{\cdot}088$ dyne-cm./radian.

With a well-annealed torsion wire, once the torsion head has been finally adjusted and fixed, the natural zero of the instrument, i.e. the reading corresponding to a gravitational field precisely uniform, may be expected ideally to display constancy with respect to time. Apart from the source of disturbance with which we shall be dealing later, the behaviour of the torsion wires is in this respect highly satisfactory, at any rate under the conditions of use in a laboratory, as will be seen. This natural zero, however, can never be observed directly, because we have no place accessible where gravity is sufficiently uniform. For even in the absence of local disturbances, the northerly gradient of gravity, except near the equator and poles, is large enough to affect the instrument—surely a striking indication of its wonderful sensitivity. Generally, therefore, the reading alters with change of azimuth of the instrument as a whole, and practice consists in observing the readings (after equilibrium is attained) with the instrument as a whole in a sufficient number of different azimuths, a graduated turntable being incorporated for the purpose.

With the gradiometer a minimum of three readings in three azimuths are

required completely to determine the gravity-gradient, and, incidentally, the natural zero of the instrument. In practice one or two more readings are taken in order to check the constancy of the zero. The author has found it convenient to throw the fundamental equation into the form:

$$n - n_\alpha = qG \sin(\alpha - \phi),$$

α G ϕ n, n_α q

where α is the azimuth of the symmetrical vertical plane of the beam with respect to an arbitrary direction (such as the magnetic meridian), G is the resultant horizontal gravity-gradient $\partial g/\partial s$, and ϕ its azimuth referred to the standard above indicated. The unknown natural zero reading is n, and n_α is the observed reading with the beam in the azimuth α. The constant q is a purely instrumental one, involving, in a manner into which we need not enter here, the distribution of the mass of the beam, the torsion coefficient, and the dimensions of the optical system.

To provide the three equations necessary to determine the unknown magnitudes n, G and ϕ, it is customary to choose values of α equal to 0°, 120° and 240° respectively. The author has found it more convenient to use four values, namely, 0°, 90°, 180° and 270°, the supernumerary one providing a convenient check on zero-constancy.

The four equations obtained are:

$$\begin{aligned}
\alpha &= 0, & n - n_0 &= -\,qG \sin \phi,\\
\alpha &= \pi/2, & n - n_1 &= \quad qG \cos \phi,\\
\alpha &= 2\pi/2, & n - n_2 &= \quad qG \sin \phi,\\
\alpha &= 3\pi/2, & n - n_3 &= -\,qG \cos \phi,
\end{aligned}$$

where n_0, n_1, n_2 and n_3 are respectively the readings observed with the instrument in azimuths 0, $\pi/2$, $2\pi/2$ and $3\pi/2$. By adding and subtracting these equations appropriately we obtain

$$n_2 - n_0 = 2qG \sin \phi,$$

$$n_1 - n_3 = 2qG \cos \phi,$$

and

$$n_0 + n_2 = 2n = n_1 + n_3.$$

The last equation provides the zero-constancy check, and the first two give the gravity-gradient in magnitude and direction. For

$$\tan \phi = (n_2 - n_0)/(n_1 - n_3),$$

and

$$G = (n_2 - n_0)/2q \sin \phi,$$

the ambiguity of ϕ being removed by reference to the last equation.

An actual example from observations at a particular location in the College laboratory will illustrate the application of these equations, and the kind of accuracy which may be hoped for under good conditions.

Date—January 5, 1932.

$$\text{Constant of instrument} = 1/2q = 10{\cdot}9 \times 10^{-9}\ \text{sec.}^{-2}$$
$$= 10{\cdot}9\,\text{E.*}$$

Time	Azimuth of instrument	Scale reading
10.30 a.m.	0°	$n_0 = 113{\cdot}9$
10.55	90°	$n_1 = 114{\cdot}5$
11.20	180°	$n_2 = 120{\cdot}1$
11.45	270°	$n_3 = 119{\cdot}4$
12.10 p.m.	0°	113·9

The last reading is an additional check for zero-constancy, which is here excellent.

The natural zero is $\quad n = \frac{1}{2}(n_0 + n_2) = 117{\cdot}0,$

or $\quad n = \frac{1}{2}(n_1 + n_3) = 116{\cdot}95,$

and the closeness of these figures again shows satisfactory behaviour.

From the above equations:

$$\tan\phi = (n_2 - n_0)/(n_1 - n_3) = -\,1{\cdot}265.$$

Hence $\phi = 128°{\cdot}3$ or $231°{\cdot}7$.

But since $n_2 - n_0$ (equal to 6·2) is positive, the equation

$$G = (n_2 - n_0)/2q \sin\phi$$

shows that $\sin\phi$ is positive. Hence $\phi = 128°{\cdot}3$ and

$$G = 6{\cdot}2/2q \times 0{\cdot}785 = 7{\cdot}9/2q = 86\,\text{E}.$$

Thus the gradient measured is 86 Eötvös units in a direction making an angle of 128° with the standard direction. It will be noted that if the readings could be always made precise and reproducible to 0·1 division, an accuracy of between 1 and 2 Eötvös units could be expected.

§3. ERRORS DUE TO ELECTRIFICATION

The above outline of the functions and use of the gravity-gradiometer enables us now to consider the special observations which form the subject of this paper. A more or less chronological order will be adopted in the description. When the instrument was first set up after the necessary repairs, it was found usually to give self-consistent results in the various stations in which it was used, but not always. More particularly immediately after the transfer from one station to another, in which the clamping and unclamping of the beam was involved, there were variations from consistency amounting to as much as 20 E. in the magnitude and 16° in the direction of the gravity-gradient determined. There were also unexplained and apparently capricious changes of the natural zero of as much as 7 scale-divisions

* "E." denotes 1 Eötvös unit, which is 10^{-9} c.g.s. unit of force-intensity gradient, i.e. 10^{-9} dyne/gm. cm., or 10^{-9} sec.$^{-2}$

E.

during one night, even when the beam was hanging free over the whole period. The cause of this rather erratic behaviour was at first assumed to be poor quality in the torsion wire, which had been similarly suspected by the Australian observers, who seem to have experienced even more marked inconsistency of the kind described. Accordingly it was decided to insert a new wire, and one tested by Dr Shaw was obtained, having a smaller torsional coefficient than the previous one.

The insertion of a new suspension, and the balancing of the beam, are matters of some delicacy. It is important, of course, that the plane of the mica ring should be parallel to the damping plate in its horizontal adjustment, and that there should be between them no hairs or dust-particles likely to interfere with free movement in the relative positions of close proximity (about 0·025 in. apart) in which they have to be used. For this reason the ring was rubbed with chamois leather, and marked electrical attraction was for a short while thereafter noticed between the ring and the damping plate. It was expected that this electrical effect would soon disappear, as, in fact, it did, so far as directly visible attractions were concerned. The erection of the apparatus was completed, apparently in a satisfactory manner, and electrification ceased to be suspected. One disturbing feature remained, however, in the final adjustment. This was the continued slow creep of the zero, which lasted several days and amounted, in all, to more than 20 degrees, before equilibrium was apparently reached. Again, this effect was—as it now appears, wrongly—attributed to the torsion wire, as an elastic after-effect due to the application of the load suspended upon it. Thereafter the behaviour of the instrument seemed to be normal, or at any rate self-consistent in repeated readings, as table 1 shows.

Table 1. Readings taken on February 29, 1932.

Time	Azimuth of instrument	Scale-reading
10.15 a.m.	0°	98·4
10.40	90°	94·2
11.20	180°	112·9
11.50	270°	117·4
12.20 p.m.	0°	98·6
12.50	270°	117·5
2.15	180°	112·8
2.45	90°	94·4
3.10	0°	98·9

Normally the readings should be constant in any particular azimuth. It will be observed that this is satisfactorily so, and that the reading in the 0° azimuth changed by only 0·5 division in about 5 hours. But the results of these observations are not even approximately in accordance with previous determinations in the same location. They give, in fact, a value of the gravity-gradient differing by nearly 100 per cent in magnitude and 20° in direction from the best earlier results.

This was the dilemma when, almost by chance, an observation was made which eventually pointed the way to the solution. The suspension had been secured with a view to testing whether the zero-creep reappeared upon release, and the damping

plate was later lowered to a position not quite so far from the mica ring as previously. It was found that the zero had changed by about 9 divisions, but was not obviously creeping. This led to systematic tests being made of the effect of other changes in the position of the damping plate, relative to the mica ring, with the surprising result that the scale-reading, and therefore the equilibrium orientation of the beam, was related in a periodic manner to the space interval between the mica ring and the plate. Moreover, the period was that corresponding to one complete rotation of the screw which controls the movement of the plate. The process of observation

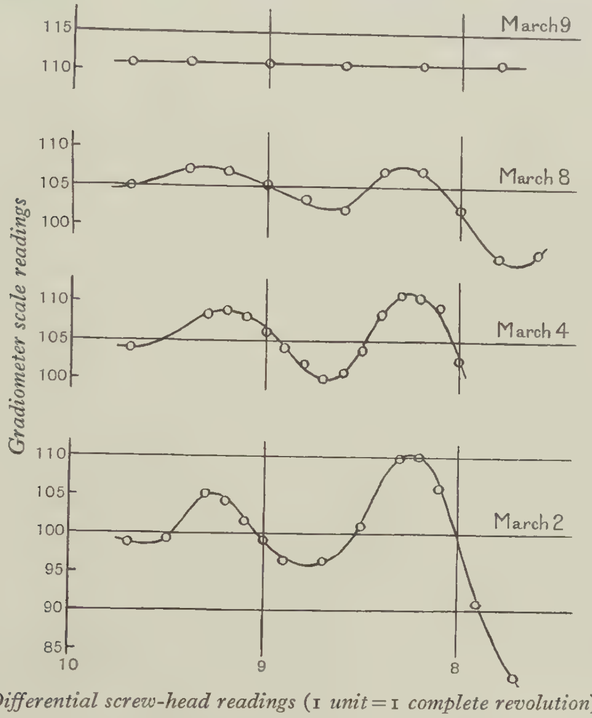

Fig. 2. Comparison of electrical effects in relation to the lapse of time, and to the introduction of ionizing material into the gradiometer.

was necessarily laborious, for a time interval of at least 15 minutes had to be allowed after each new adjustment for equilibrium to be attained. Yet the relation indicated was quite definite, and readings could be closely repeated, especially if taken after screw-rotations always in the same sense.

The results are shown graphically in the lowest curve in figure 2. The abscissae are the screw-head readings, which decrease as the damping plate rises, and a change of reading of unit corresponds to one complete revolution of the screw-head. The form of the curve obviously suggests the operation on the beam of a horizontal force-moment which increases as the damping plate approaches the mica ring, and also fluctuates considerably with the rotation of the driving-screw. Electrification was again suspected; a charge localized on a comparatively small

region of the mica, accompanied by a periodic relative tilt of the damping plate owing to imperfect travel of the actuating screw, would account qualitatively for the observations. And this is, indeed, the explanation. But at the time it was difficult to believe that a charge could remain insulated so persistently, even though the weather at the time was very dry; for the mica ring had been already suspended freely for several days—from February 29 to March 2, without any contact which might have produced new electrification. Moreover, it was found that the effects could be repeated without much change on subsequent days. Thus, for example, the curve labelled March 4 in figure 2 shows a little reduction of amplitude, but not much.

Still, after seeking in vain for another explanation, such as gravitational asymmetry in some part of the relatively rotating screw, it became necessary to adopt the Sherlock Holmes principle that, the impossible having been eliminated, what remains, however improbable, must be the truth. Accordingly the instrument was opened up, and the mica ring was subjected for about an hour to the radiation from a fairly active preparation of radium D which happened to be available. The ionizing effect of the α-particles operated chiefly upon the air above the mica, as it was not convenient with this preparation to ionize the air between the mica and the damping plate, except indirectly by slow diffusion. But evidently some effect had been produced, for on the following day, March 8, the curve so labelled in figure 2 was obtained. Here, it will be seen, the amplitude is definitely less, although the effect still definitely persists. As a more effective ionizer about $\frac{1}{3}$ mgm. of meso-thorium was introduced into the apparatus, and about one hour afterwards the observations in the topmost curve of figure 2 were taken. This shows very strikingly the complete elimination of the effect, which must accordingly be attributed to an electric charge on the mica, unable to escape owing to the high insulating properties of this material.

It is impossible to say whether this charge was the remains of that originally developed on the mica while it was being cleaned with leather, although probably it was so; for in the meantime the mica had been at least once in contact with the damping plate, and this might have disturbed the earlier electrical distribution. But the fact remains that a small charge (an estimate of its amount is made later) persisted nearly unchanged in amount and location on the mica for more than a week. For while the constancy of amplitude in the earlier curves implies little variation in the amount of the charge, the constancy of the positions of the maxima and minima equally implies its stationary position.

It is interesting to examine a little more closely the mode of operation of this electrical effect, now happily eliminated, and to consider its possible bearing upon normal gravity determinations made with the instrument. The complete removal of the electrification made known precisely for the first time the reading, 111·0, corresponding to the equilibrium of the suspended system under the sole action of the torsion in the wire and the differential gravity forces. Comparison of this with the readings indicated in any of the lower figure 2 curves shows that the effect of electrification was apparently two-fold, first, a deflection (in the sense of

reduced reading) which became notably greater as the damping plate approached the mica, and, secondly, a fluctuating deflection, correlated to the azimuth of the damping plate screw-head. This latter deflection appeared to be superimposed upon the first as a pseudo-zero-line, and evidently increases in amplitude as the distance between the mica ring and the damping plate is reduced. An attempt has been made to indicate the dissection of the curves in figure 3. The x-axis itself is taken to represent the undisturbed zero reading; the differences between this and the other readings are the ordinates, which thus become positive; and the abscissae are the azimuths of the screw-head itself (instead of the readings on its drum) reckoned from its angular position corresponding to contact between the damping plate and the mica. These abscissae are thus also proportional to the distance between the mica and the plate. Actual deflections are represented by the full curve, and the dotted line is a guess at the changing zero about which the fluctuations seem to occur.

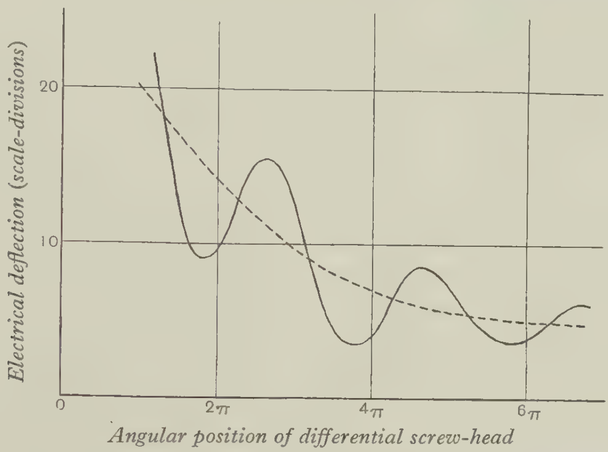

Fig. 3. Electrical effect of rotation of differential screw-head. The angular position represented of the screw-head is proportional to the distance between the mica ring and the damping plate, 2π being equivalent to 0·025 in.

A satisfactory explanation of the electrical effect can be made on this basis. In order to produce an angular deflection at all the charge must be acted upon by a force which has a horizontal component, for a precisely vertical force would produce no moment about the vertical axis of the system. Moreover, since the charge on the mica was always very close to the relatively extensive brass damping plate (the maximum separation was less than 0·1 in.) it is reasonable to suppose that practically all the lines of force terminated normally upon this plate. This implies that the plate itself was never truly horizontal. The evidence provided in figure 3 that, on the average, the force increased notably with diminution of the plate distance, also suggests strongly that the charge was very much localized, in comparison with the separation. The explanation of the effects accordingly offered is that a charge on the mica, practically constant in amount and confined to a comparatively small area, was attracted by the damping plate with a force which

always had a horizontal component, owing to this plate being inclined to the horizontal. This inclination varied from its mean value owing to imperfect action of the differential screw, the variation being repeated for each complete rotation of the latter. Consequently, the effective part of the plate (which does not itself rotate bodily)—i.e. that immediately underneath the charge—executed periodic changes of inclination, and thereby varied periodically the magnitude of the horizontal force-component. The total effect would, of course, be a mixed one, combining the results of inclination and proximity. If, for example, the screw had been perfect in its action, so that the damping plate maintained a constant inclination, we should have expected simply a gradual growth of the deflection as the plate rose, owing to the increase in the magnitude of the electrical attraction—some such curve, in fact, as the dotted line in figure 3. On the other hand, if the charge had been distributed over so great an area that the magnitude of the attraction was independent of plate distance, the periodic error of the screw would have given rise to deflections fluctuating with constant amplitude about a constant mean. The changing amplitude, together with the changing mean, point clearly to a localized charge on the mica, combined with imperfect screw action, as the cause of what has been observed.

§4. QUANTITATIVE ESTIMATE OF ELECTRIFICATION AND PLATE-TILT

It is easy now to think of further tests that might have been made to render the matter more certain. The feeling of relief that came with the disappearance of the electrification after several weeks' work, and with the anticipation that the gradiometer would behave better, was soon replaced by some regret that the same charge could not be re-established for further experiments. It is nevertheless possible to carry the argument a little further, and make an estimate of what was in all probability a not very different charge, and of its effect on gradiometer observations normally carried out. The observations recorded in table 1 were made at a time just previous to the discovery of the electrical effect, and the mica was undoubtedly then charged also. In the interval the mica ring had rested (it was not clamped) for 50 minutes upon the damping plate. This may, of course, have changed the charge both in magnitude and location, but the little evidence of comparative readings which is available makes it fairly certain that the charge before the interval was in the same position and substantially the same in amount as it was afterwards. At any rate the charge was what had survived after six days of free suspension, and was not enough to make the behaviour of the gradiometer appear other than normal.

We can use these observations in combination with corresponding ones obtained when the mica had been completely discharged, to form an estimate of the mean tilt of the damping plate and the magnitude of the charge. In these observations, of course, there is no question of variation of plate-distance, which is constant at the normal damping position. Any electrical effect is due solely to variation of

plate-inclination with change of azimuth of the instrument as a whole. The comparative readings are given in table 2, in which are recorded also the limits of free swing, for reasons which will appear later.

Table 2

Azimuth of instrument	Gradiometer readings		Limits of free swing
	(*a*) Previous to discharge	(*b*) After discharge	
0°	98·4	111·0	76–135
90°	94·2	112·2	77–136
180°	112·9	122·0	73–132
270°	117·4	120·6	72–131
0°	98·6	110·9	

Both sets of readings are satisfactory as regards repetition in the same azimuth, but their wide and inconsistent differences are due to the electrification in the first case. If they are used to calculate the apparent gravity-gradient, we obtain:

(*a*) 166 E. at azimuth 148°.

(*b*) 83 E. at azimuth 127°.

The latter, of course, is the correct result, and, as is evident, the former differs from it by 100 per cent in magnitude and 21° in direction. This large difference must be attributed to the axis of rotation of the instrument not being vertical, for otherwise the relative inclinations of the mica ring and damping plate would not change, and electrification could do no more than produce a constant deflection, which would disappear in the calculation. In other words, spurious results would be avoided, even with an electrified system, if the axis of rotation were truly vertical. But it turns out that the axis would have to be vertical with impracticable precision.

The levels provided on the instrument are such that one division on the bubble scale corresponds to about 20 seconds of arc, but they are difficult to adjust so as to use fully this sensitivity, which, indeed, is quite unnecessary in normal practice. They were not, in fact, read very carefully while the instrument was in the adjustment with which we are at present concerned. But it happens that some incidental and not very accurate readings, namely, those corresponding to the limits of free swing shown in table 2, provide a relatively sensitive method of estimating the extent of the departure of the rotation axis from the vertical. If this axis were truly vertical, rotation of the instrument would bring about no relative alteration of the positions of its various parts, including the suspension, and contact of the latter with the stops would occur always at the same readings. The fact that these limiting readings did not remain constant accordingly implies that the axis of rotation was not quite vertical; and the change of the limits with azimuth, used in combination with the geometry of the instrument, shows that the error of adjustment was approximately 30 seconds of arc. Thus, during a complete rotation, the

damping-plate tilt altered through a maximum range of just about 1 minute—a change of angle entirely negligible in the normal gravitational use of the apparatus, but responsible for a large part of the spurious electrical effect.

It turns out that besides this periodic electrical disturbance associated with azimuth there was a constant effect due to the plate having a small permanent tilt upon which the variable part during rotation was superimposed. If we take the differences between corresponding readings in the (*b*) and (*a*) columns of table 2, we isolate from the true gravitational control the whole of that due to plate tilt and electrification. These differences, which correspond to electrical deflections in the sense chosen for positive azimuth, are given in the second column of table 3.

Table 3

Azimuth (α)	Electrical deflection (scale-divisions)	
	Observed	Calculated
0°	12·6	12·5
90°	18·0	18·1
180°	9·1	9·0
270°	3·2	3·3
360°	12·6	12·5

If our explanation of the effect of plate tilt is correct we should expect these observations to be representable by an equation of the form

$$d = a + b \sin(\alpha + \epsilon),$$

d, α, a, b with d as the electrical deflection, and α as the azimuth of the instrument; a, b
ϵ and ϵ being constants. Although the test is not very stringent, there being only one observation in excess of the number of constants, the equation

$$d = 10{\cdot}7 + 7{\cdot}6 \sin(\alpha + 13^\circ{\cdot}3)$$

does in fact fit all the points with a precision equal to the accuracy of reading, namely, 0·1 division. We may therefore take the curve shown in figure 4 as re-

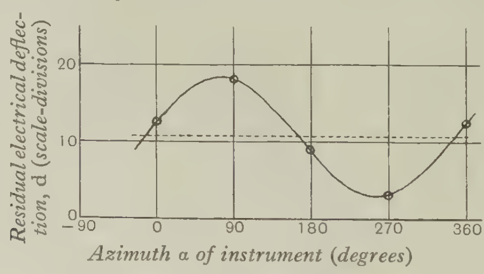

Fig. 4. Electrical effect of change of azimuth of instrument. Calculated curve $d = 10{\cdot}7 + 7{\cdot}6 \sin(\alpha + 13^\circ{\cdot}3)$; observed points denoted by circles, and zero of sine curve by dotted line.

presenting by its ordinates the course of variation of total plate tilt with azimuth. This may be dissected into the constant part, 10·7, and the variable part which has an amplitude of 7·6. The former corresponds to the permanent part of the

plate tilt and the latter to its change with azimuth. We have already seen that the variable part had an amplitude of 30″, and we can now conclude that permanent part was $30 \times 10{\cdot}7/7{\cdot}6$ or 40″ approximately—again an angle of no importance in gravitational practice with the instrument*.

This estimate of the magnitude of the plate tilt provides means of making a rough estimate of the amount of electrical charge which was operative, if certain assumptions are made as to its distribution. The distance between it and the plate was 0·025 in. or 0·064 cm., and it is difficult to believe that the charge was confined to a region on the mica small compared with this, although the evidence points to its not being large. With this reservation, and assuming nevertheless that the charge was effectively at a point on the mica about midway between the inner and outer edges of the ring, we can calculate its amount, for we know the change of scale-reading, and, therefore, the torsional force-moment, corresponding to a given plate tilt. Thus for any azimuth we equate the horizontal component of the force on the electric charge (this force itself being always normal to the plate) multiplied by its distance from the axis of suspension, to the residual torsion in the wire, as indicated by the difference between the scale-readings under charged and uncharged conditions. The result obtained is that the charge was about 0·07 e.s.u. This is certainly an under-estimate, for the assumption of any distribution of the charge wider than a point leads to a higher value. For example, if we suppose it to be spread uniformly over 1 $cm.^2$ the amount becomes three times as great. Equally certainly this is an over-estimate, for the observations show a marked dependence of attractive force upon distance, even at small separations of the order of 1 mm., between the mica and brass surfaces. It is safe, therefore, to take it that the charge was of the order of 0·1 e.s.u.

§ 5. PRACTICAL CONCLUSIONS

This charge was much too small to make itself evident except in such an extremely delicate system as the gradiometer. Indeed, its marked disturbing effect is a tribute to the sensitivity of this nevertheless portable and robustly constructed instrument. The important practical point is, however, that electrical charges, small even in comparison with that above calculated, must be eliminated whenever formed. That they can be produced by the contact between the mica and brass, necessary during transportation of the instrument, has been shown by subsequent observations.

It is only rarely that the effect becomes obvious, but when it is remarked that on more than one occasion the release of the suspended system has been followed by a period of erratic and unstable behaviour lasting two hours, even with the meso-thorium within the instrument, it will be realized that less violent and obvious effects are likely to be much more frequent. In fact, a very small charge might be

* It is perhaps worthy of note that a new adjustment of the levelling, made subsequently with greater care, resulted in the limit readings becoming practically independent of the azimuth. This showed that the axis of rotation had been rendered much more truly vertical.

much more insidious than a large one, for it would probably fail to arouse suspicion and consequently lead to spurious gravitational results.

Safety in this respect lies in making provision either for the prevention of the development of all charges of electricity, however small, or for their rapid dissipation. The substitution of some suitable metal for the mica would effectively fulfil the former alternative, because the whole apparatus would then be of conducting material, and its inner parts could sustain no electrification. Or the mica might be coated with metal, for example by spluttering; but this would probably not be satisfactory or permanently effective, owing to the liability of the metal to be rubbed off by contact with the brass clamps. The author, however, favours the second alternative, namely, the use of sufficient ionizing material suitably disposed to dissipate quickly any charges formed. The mica has several features in its favour—its flatness and lightness, for example—and ionization has been proved a reliable means of getting rid of the electrification, whether this is produced during the original setting up of the apparatus or by the contacts inevitable in normal use.

Since the introduction of the meso-thorium the instrument under discussion has been a joy to use, and its performance has more than realized expectations. It seems almost certain that, when it was first set up with the new suspension, the considerable initial electric charge on the mica disappeared much more slowly than one would have anticipated, its gradual reduction of control giving rise to the prolonged creep of the equilibrium reading. Even after apparently normal equilibrium had been reached there remained a relatively small but insidious charge, which would have taken weeks to diminish by leakage to a really negligible amount. The insertion of the meso-thorium greatly accelerated this process, and its presence has continued thereafter as an effective safeguard. And, except on those rare occasions already mentioned when the temporary erratic behaviour was too self-evident to be misleading, the observations with the instrument have been extraordinarily self-consistent in spite of frequent use of the clamping device. Deliberate attempts have been made to facilitate electrification by encouraging friction between the mica and its clamps. But in practically every case its effect, if it was ever produced, had disappeared before the normal minimum time after release for taking a reading—about 25 minutes.

As illustrating the very marked improvement in the performance of the instrument in the absence of electrification, the results shown in table 4 and obtained in

Table 4

Date	G (Eötvös units)	ϕ (degrees)
March 10	83	127·3
,, 14	85	127·7
,, 15 (1)	82	126·3
,, 15 (2)	83	126·7
April 1 (1)	81	127·0
,, 1 (2)	82	127·0
Mean	83	127·0

the same place on different occasions, for the magnitude (G) and the azimuth (ϕ) of the gravity-gradient, are worth noting.

The extreme variation of the magnitude is only 4 E. and of the direction only 1·4 degrees. These may be compared with the errors of 83 E. and 21° for which electrification was on one occasion responsible. Moreover, the equilibrium reading of the instrument in a fixed azimuth has not been observed to vary by as much as 1 division for a whole month, although the clamping device has been used frequently during this period. When it is remembered that constancy in this respect is only demanded for rather less than two hours—the maximum time normally occupied at a station—the present reliability of the instrument becomes apparent. It seems reasonable, after the experience described with this one instrument, to doubt whether all the field observations so far made with gradiometers have been sufficiently free from the spurious effects of electrification. In some cases, probably only a few, quite large errors may have been involved, but there has been the danger that many of the results have been at least slightly tainted. The uncertainty in this matter arises from the way in which the electrical effect may remain hidden, allowing results which are really vector mixtures of the true and the spurious to pose as genuine gravity-gradients. A too variable natural zero should, perhaps, have aroused suspicion, as, indeed, it sometimes has done; but the elastic properties of the torsion wires have had to bear the blame. The torsion wire has now been found not guilty, at least of the chief indictment; and the real culprit has been detected and abolished, or, rather, deported. It is hoped and believed that this example will be followed, not only in newly manufactured gradiometers, but in those also which have already served in the field. For the means of eradication is both simple and inexpensive, and its application will without doubt enhance still more towards perfection the accuracy and reliability of the already wonderful performance of these instruments.

DISCUSSION ON THE PRECEDING TWO PAPERS.

Dr H. Shaw. The author has obtained some very simple and convenient formulae for the expression of these gravitational effects. His method of completely separating the two portions of the horizontal couple will be most helpful to the student in enabling him to form a better mental picture of exactly what is happening in the instrument. With regard to the graphical method put forward in the paper I am inclined to be a little hesitant, for although its simplicity is obvious in the laboratory, its application in the field would I fear be somewhat more difficult. However I should like to see it tested under field conditions, against the methods at present in operation, and if it proves quicker and more convenient, there is little doubt that it will be applied extensively.

With regard to the second paper, I would say that the metal disc suggested by Prof. Rankine was tried before mica was adopted, and was discarded on account of the difficulty of securing a ring which is at the same time flat, light and rigid. Ordinary mica too was soon found to be useless, and even ruby mica, apparently quite free from intrusions, was often found to possess impurities which in the earth's magnetic

field gave deflections of from 10 E. to 40 E. It will be seen therefore that it has proved to be a matter of some difficulty to obtain mica which is up to the standard required for these instruments.

With reference to the electrical troubles which form the subject of this paper, it is very curious that prior to Prof. Rankine's experiments we had not encountered anything at all comparable to the disturbing effects he describes, and there can be little doubt that the trouble is due, at any rate in part, to a too energetic use of the chamois leather. I was reminded this morning that Prof. Boys warned us of this (and other) troubles in 1921, when the Eötvös torsion balance was first exhibited and described at a geophysical discussion of the Royal Astronomical Society.

I would like to ask the author whether the long zero-creep referred to on pp. 479, 480 was upward or downward, and whether it may be attributed wholly or only in part to the gradual leakage of the electrical charge. With regard to the possibility of errors in the field observations so far made with gradiometers, I would say that the constancy of the natural zero is perhaps one of the observer's main safeguards. It happens however that this point has long been recognized as an important indicator of the efficient operation of all instruments of the Eötvös torsion balance type, and so far as our own field operations are concerned it has always been kept under close observation, so that I have no fears. It may be of interest to note that the first gradiometer, which has now been in operation for over four years, is still in the field, where it continues to give the most gratifying results.

Users of this instrument are much indebted to the author, firstly for discovering and tracking down this source of danger, secondly for determining its nature, and lastly (what is perhaps even more important) for showing how it may be completely eliminated.

Mr E. Lancaster-Jones. The treatment outlined in the first paper will greatly assist students who appreciate geometrical or graphical illustrations and dislike analysis. The ordinary analytical treatment, using axial components of the gravity magnitudes, has however the merit that the component effects are simple algebraical numbers, and therefore can easily be summed by unskilled operators or by mechanism. At the same time there is a lot to be said for plotting the terrain and topographical-correction quantities, as a check on numerical calculations.

As regards the second paper, all users of the gradiometer will be greatly indebted to Prof. Rankine for this investigation. It has revealed a hitherto unsuspected source of trouble, and indicated the cure. It is highly improbable, fortunately, that previous field-work results obtained with the gradiometer have been subject to unsuspected errors by reason of electrical effects of this type. Readings have always been repeated and zero-fluctuations carefully scrutinized. Had electrical effects been present, they must surely have revealed themselves by reason of the instability of the zero and non-repetition of readings in a given azimuth. In practice, the instrument is supported on footplates bedded usually in soft ground, and a certain amount of "settlement" is inevitable; this causes the level of the instrument to vary sufficiently to accentuate the electrical effects very considerably.

Also, in most of the practical surveys so far undertaken with the gradiometer the results have been checked by using other instruments at intervals, and very few discrepancies have been revealed, except in the Australian work. The ionization treatment advocated by Prof. Rankine should certainly be adopted, but it may be as well to consider methods of clamping the beam at its metal parts, leaving the mica disc "in the air."

Prof. W. Wilson. The author's vectorial methods are very valuable, even though they may not be the most suitable for purposes of computation in the field. I would suggest that the horizontal directive tendency is a tensor of rank 2.

Dr D. Owen. It seems almost incredible that in an ordinary damp atmosphere electric charges should persist on the surface of mica for a period of a fortnight. I should like to suggest that charges may occur in the interior of the mica sheet owing to shearing produced in the course of the adjustment. The degree of non-homogeneity in mica seems to warrant the possibility of such an effect.

Dr J. E. R. Constable. I was very interested in the author's account of the trouble which he experienced owing to charges on the mica disc, as some time ago, while working with a Geiger counter, I experienced considerable trouble due to the same cause. A Geiger point counter fitted with a mica window rapidly ceased to function when exposed to β and γ radiation. Touching the surface of the mica with an earthing wire always restored the activity at once. In view of the fact that I was using up to 2000 volts between the case of the counter and its inner electrode it would seem that potential-differences of this order can be generated by the charges on the mica. These charges only appeared when the counter had the potential-difference applied to it, so that Prof. Rankine's method of removing the charges by mesot-horium radiations should be quite satisfactory provided there are no electric fields in the neighbourhood. To secure the highest efficiency it might therefore be advisable to enclose the meso-thorium in an earthed metal container.

Mr J. H. Awbery said that the quasi-vectorial diagram in figure 2 of the first paper reminded him of diagrams used in the theory of finite rotations, the plane of figure 2 corresponding to the spherical surface dealt with in the theory.

Mr J. Guild asked whether a paper disc could be substituted for the mica disc which had proved so troublesome.

Mr T. Smith. As regards the first paper: as a rule I much prefer working with rectangular to polar coordinates, but comparison of equations (1) and (6) of the paper on gravity surveys makes it quite apparent that the latter are much to be preferred here. In field work the instrument is set by means of an angular scale, and the solution given in equations (11) is the simplest and most direct that can be expected. The use of graphical methods is a distinct question. Personal preferences may carry some weight here. As an accuracy of about 1 per cent is sufficient, graphical methods are certainly admissable and will probably be more quickly carried out than the resolution required by the alternative method. The construction for R may be compared with that for the combination of two astigmatic lenses.

As regards the second paper: the paper on the gradiometer is a most interesting account of the way in which trouble with an instrument was tracked to its source. Dr Shaw has told us that Prof. Rankine was misusing the instrument in rubbing the mica ring with the chamois leather, but this misuse, in combination with somewhat imperfect action of the screw, has been the means in Prof. Rankine's hands of entirely altering the value of the instrument. I can only hope that the author will always be as fortunate in his mishandling, more particularly if it frequently results in the presentation of such an interesting paper to the Physical Society. Like all who have listened to his explanations I would like to express my appreciation of the very lucid way in which both papers have been presented to us.

Author's reply. With regard to the first of these two papers I welcome the expression of views by two such experienced workers with torsion balances as Dr Shaw and Mr Lancaster-Jones. I agree that the ultimate test of the relative merits of the customary analytical method, as contrasted with the graphical method now proposed, is in respect of rapidity and convenience in the field work. My students and I are relatively inexperienced in this aspect of the work, but we hope gradually to accumulate data to render the comparison possible.

I am not sure that I understand the terms used by Prof. Wilson, but, whatever the horizontal directive tendency is called, I have recognized that the graphical rules for its addition suggested other similar directed quantities of higher orders. So far I have resisted the temptation to investigate these mathematically, because their practical application was not obvious. It is interesting to note that the graphical rules for addition of the R-values are similar to those applicable in the theory of finite rotations, as indicated by Mr Awbery, and in the theory of the combination of astigmatic lenses, as Mr T. Smith has mentioned.

On Mr T. Smith's other remarks I wish to say how gratified I am to have for my mode of transformation of the equations the approval of so penetrating and reliable a mathematician. This is a real encouragement to go on further in the direction indicated, and I shall do so.

The remarks of Dr Shaw and Mr Lancaster-Jones on my second paper are also valuable as showing the infrequent incidence of marked electrical effect in the many surveys already carried out with gradiometers. But I do not think it is safe to assume, as Dr Shaw apparently does, that strong electrification of the mica is always due to too vigorous rubbing during cleaning. Considerable charging occasionally occurs as a consequence of the normal clamping and releasing of the beam, and one cannot be sure that small effects do not occur quite frequently. In reply to Dr Shaw's question, I would say that the zero-creep was in the direction corresponding to increasing reading, i.e. a rotation in a counter-clockwise sense as viewed from above the mica ring. I am convinced that the zero-creep observed was almost if not wholly due to leakage away of the original electric charge, for, after provision had been made for rapid discharge, the zero was very constant, in spite of repeated alternations of loading and unloading of the suspension. I agree with Dr Shaw and Mr Lancaster-Jones that constancy of repeated readings would be a

satisfactory test of the absence of electrification, but constancy over many hours would be required, because of the slow rate of leakage. As I have indicated in the paper, repetition of readings was apparently satisfactory over the normal observation-period, even when the mica was strongly charged. On the other hand, the "settlement" to which Mr Lancaster-Jones has referred, by varying the axis of rotation in field use, would provide a sure criterion.

In reply to Dr Owen I should remind him that actually the observations were made in very dry weather. Apart from this, however, I do not think the charges could be internal, for, if so, neither the positive nor the negative could leak away, and their combined effect would be nil. Also, the quick disappearance of the charges in ionized air suggests external location on the mica, because diffusion into the mica sheet would presumably be very slow.

Dr Constable's experience with the Geiger counter confirms the view that one has to be very careful of electrical effects on insulators. My own recent observations have, in fact, made me suspicious of all insulated suspensions, or suspensions composed wholly or partly of insulating material.

The inventors of the gradiometer would be better able than I to answer Mr Guild's question; but Dr Shaw's remarks would seem to imply that paper could not be made, or ensured to remain, flat enough to serve for damping purposes. Mr Lancaster-Jones's suggestion to clamp the system by its metal parts and leave the mica always free is ingenious, but I think ionization is safer.

Mr T. Smith emphasizes an interesting point, the implications of which are somewhat curious. It is true that it was the imperfection of the differential screw which led to the charging being discovered and eliminated. I doubt very much whether the spurious effect could have been distinguished otherwise from normal gravity effects, for the manipulation of the damping plate formed the crucial test. Does this imply that sometimes imperfect workmanship is to be not only tolerated but approved?

THE FALL OF POTENTIAL IN A CHARGED INSULATED CABLE

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ABSTRACT. The expansion theorem of Heaviside is applied to the solution of the problem in which a cable, having been charged until it reaches its steady state, is insulated at the sending end; and the potential is required at any point and at any time after insulation. It is found that at the sending end there is an initial steep fall of potential which is due to a part of the charge being drained away from this end, in order to equalize the potential throughout the line when the exciting source has been removed. After the potential has become uniformly distributed, it falls with time according to a simple exponential law.

§ 1. INTRODUCTION

V_0 WHEN a condenser with a good solid dielectric between its plates is charged to a potential V_0 and then left insulated, it is easily shown that V_t, t the potential V_t at any time t after insulation is given by

$$V_t = V_0 e^{-Gt/C},$$

G, C where C is the capacity of the condenser and G is the leakance or the reciprocal of the dielectric resistance. Use has been made of this formula to deduce the resistance of the dielectric when the capacity of the condenser is known. The method is a very old one, and is believed to be due to Latimer Clark (*c.* 1867) who applied it to some of the early submarine cables. Fleeming Jenkin(1) described a method of measurement in which an "inferred zero" was used, and we find the method advocated for cable-testing by various writers since that time.

A long cable is, however, not exactly equivalent to a condenser. We have to consider the fact that the capacity and leakance are distributed throughout its length. The aim of the present investigation is to find an expression for the potential at any point along the conductor of a cable which has been charged and then insulated, the far end being free.

§ 2. HEAVISIDE'S EXPANSION THEOREM

This very useful theorem was stated without proof in Heaviside's *Electromagnetic Theory*(2). Recently Vallarta(3) has reconstructed the argument from Heaviside's scattered writings. In many cases in which a steady force is discontinuously applied or removed, the expansion theorem leads quickly to a solution which could only be obtained otherwise by Fourier integrals.

The method of working may be illustrated by considering a circuit of impedance Z which has a steady e.m.f. V_0 suddenly impressed upon it. The current i is given by Z, i

$$i = V_0 / f(p) \qquad \text{......(1)},$$

where $f(p)$ is a function known as the "operational-impedance." This is derived from the ordinary impedance Z to alternating current by the substitution of the Heaviside operator p (or d/dt) for the imaginary operator $j\omega$. The expansion theorem then states that $f(p)$

$$i = V_0 \left\{ \frac{1}{f(0)} + \sum_p \frac{e^{pt}}{p f'(p)} \right\} \qquad \text{......(2)},$$

where the value or values of p are obtained by equating $f(p)$ to zero. The steady term $f(0)$, obtained by putting $p = 0$ in $f(p)$, is the resistance to direct current remaining when all transients have died away.

§ 3. APPLICATION TO A CABLE WITH UNIFORMLY DISTRIBUTED CONSTANTS

The line is assumed to be of length l and to possess resistance, capacity and leakance R, C and G per unit length. The inductance is assumed to be negligible. l R, C, G

With the far end free, a potential V_0 is applied at the sending or head end of the line, and after the steady state has been attained the head end is insulated. The potential $V_{x,t}$ at any point distant x from the head end at any time t is required. $V_{x,t}, x, t$

The steady-state current into the line is given by the well-known expression

$$i_0 = V_0 \sqrt{(G/R)} . \tanh l \sqrt{(RG)} \qquad \text{......(3)}.$$

Now the action of freeing the cable is equivalent to impressing an equal and opposite current on the line at the head end, the other conditions remaining the same. The potential at any point x will then be expressed by

$$V_{x,t} = V_{x,0} - V_x' \qquad \text{......(4)},$$

where $-V_x'$ is the effect due to the current $-i_0$ at the head end. $V_{x,0}$ is the potential at the point x at the moment of freeing the cable.

From transmission theory(4) we have

$$V_{x,0} = V_0 \cosh x \sqrt{(RG)} - i_0 \sqrt{(R/G)} \sinh x \sqrt{(RG)} \qquad \text{......(5)},$$

but

$$i_0 \sqrt{(R/G)} = V_0 \tanh l \sqrt{(RG)}.$$

Therefore

$$V_{x,0} = V_0 \frac{\cosh (l - x) \sqrt{(RG)}}{\cosh l \sqrt{(RG)}} \qquad \text{......(6)}.$$

Again(5)

$$V_x' = V_0 \frac{\cosh P(l - x)}{\cosh Pl} \qquad \text{......(7)},$$

but

$$V_0 = i_0 Z_0 \coth Pl.$$

Therefore

$$V_x' = i_0 Z_0 \frac{\cosh P(l - x)}{\sinh Pl} \qquad \text{......(8)},$$

where

$$Z_0^2 = R/(G + pC) \qquad \text{......(9)}, \qquad Z_0$$

and

$$P^2 = R(G + pC) \qquad \text{......(10)}. \qquad P$$

Substituting for i_0 from (3),

$$V_x' = \frac{V_0\sqrt{(G/R)}\tanh l\sqrt{(RG)}}{f(p)} \qquad \ldots\ldots(11),$$

where

$$f(p) = \frac{P}{R}\cdot\frac{\sinh Pl}{\cosh P(l-x)} \qquad \ldots\ldots(12).$$

Putting $p = 0$ in (12) we have

$$f(0) = \sqrt{(G/R)}\cdot\frac{\sinh l\sqrt{(RG)}}{\cosh(l-x)\sqrt{(RG)}} \qquad \ldots\ldots(13).$$

Converting (12) to circular functions,

$$f(p) = -\frac{jP}{R}\cdot\frac{\sin jPl}{\cos jP(l-x)} \qquad \ldots\ldots(14).$$

Equating to zero,

$$\sin jPl = 0,$$

$$P^2l^2 = -m^2\pi^2 \qquad \ldots\ldots(15),$$

where m is zero or any integer.

Substituting from (10),

$$p = -(m^2\pi^2 + GRl^2)/CRl^2 \qquad \ldots\ldots(16).$$

Differentiating (14) with respect to P and substituting from (15),

$$\frac{d}{dP}f(p) = \frac{Pl}{R\cos(1-x/l)\,m\pi}\left[\frac{\sin m\pi}{m\pi} + \cos m\pi\right],$$

but $2PdP = CRdp$ from (10). Therefore

$$f'(p) = \tfrac{1}{2}Cl\sec(1-x/l)\,m\pi\,[\cos m\pi + \sin m\pi/m\pi] \qquad \ldots\ldots(17),$$

$$\frac{1}{pf'(p)} = -\frac{2Rl\cos(1-x/l)\,m\pi}{(m^2\pi^2+GRl^2)(\cos m\pi + \sin m\pi/m\pi)} \qquad \ldots\ldots(18).$$

Hence, combining (11), (13), (16) and (18) in the expansion formula, we have

$$V_x' = V_0\sqrt{\frac{G}{R}}.\tanh l\sqrt{(RG)}\left[\sqrt{\frac{R}{G}}\cdot\frac{\cosh(l-x)\sqrt{(RG)}}{\sinh l\sqrt{(RG)}}\right.$$

$$\left. -2Rl\sum_{m=0,1,2\ldots}\frac{\cos\left\{\left(1-\frac{x}{l}\right)m\pi\right\}.\exp\left(-\frac{m^2\pi^2+GRl^2}{CRl^2}.t\right)}{(m^2\pi^2+GRl^2)(\cos m\pi+\sin m\pi/m\pi)}\right]$$

$$= V_0\left[\frac{\cosh(l-x)\sqrt{(RG)}}{\cosh l\sqrt{(RG)}} - 2l\sqrt{(RG)}.\tanh l\sqrt{(RG)}\right.$$

$$\left.\times\sum_{m=0,1,2\ldots}\frac{\cos\left\{\left(1-\frac{x}{l}\right)m\pi\right\}.\exp\left(-\frac{m^2\pi^2+GRl^2}{CRl^2}.t\right)}{(m^2\pi^2+GRl^2)(\cos m\pi+\sin m\pi/m\pi)}\right] \qquad \ldots(19).$$

Finally from (4) and (6) we have

$$V_{x,t} = 2V_0 l\sqrt{(RG)}\tanh l\sqrt{(RG)}\sum_{m=0,1,2\ldots}\frac{\cos\left\{\left(1-\frac{x}{l}\right)m\pi\right\}\exp\left(-\frac{m^2\pi^2+GRl^2}{CRl^2}t\right)}{(m^2\pi^2+GRl^2)(\cos m\pi+\sin m\pi/m\pi)} \qquad \ldots\ldots(20).$$

This is the required expression for the potential at any point at any time after

freeing. To obtain the potential at the head end, x is made equal to zero, and we find

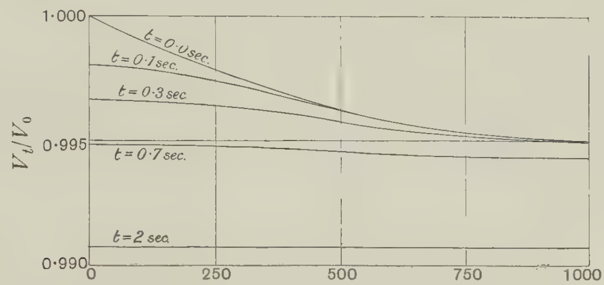
$$V_{0,t} = 2V_0\, l\sqrt{(RG)}.\tanh l\sqrt{(RG)} \sum_{m=0,1,2\ldots} \frac{\exp\left(-\dfrac{m^2\pi^2+GRl^2}{CRl^2}t\right)}{(m^2\pi^2+GRl^2)\,(1+\tan m\pi/m\pi)} \qquad \ldots\ldots(21).$$

§ 4. NUMERICAL EXAMPLE

In figure 1 is shown the potential at various times after freeing at any point in a cable 1000 miles long, having the following constants per mile:

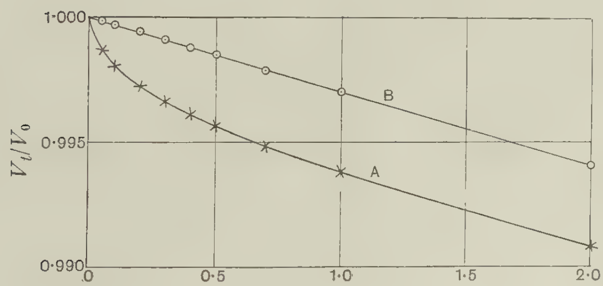
$$R = 10\Omega, \quad G = 10^{-9}\text{ mho}, \quad C = \tfrac{1}{3}\mu\text{F}.$$

It is seen that the potential at the head end begins to fall immediately, while at a point further down the line the potential remains steady for an appreciable time. In this particular case the potential distribution is practically uniform after two seconds.



Distance from sending end (miles)

Fig. 1.



Time t (sec.)

Fig. 2.

Curve A in figure 2 shows the fall of potential with time at the head end of this same cable, calculated from equation (21). Curve B in the same figure was obtained by putting R equal to 0 in (20) or (21) so that

$$V_t = V_0\, e^{-Gt/C} \qquad \ldots\ldots(22),$$

which, as was pointed out in § 1, means that we have neglected the distributive effect and treated the cable as an ordinary condenser. This curve is shown for comparison with the more accurate expression (21).

§ 5. CONCLUSION

The expression obtained for the fall of potential at the head end of the cable shows that there is an initial sharp decline which is not predicted by the approximate formula (22). This effect is due to the redistribution of the charge along the cable, tending to uniformity throughout, when once the source has been removed.

In measuring the insulation resistance of a long cable by the loss-of-charge method, the higher potential value is commonly taken to be that of the charging battery, and the lower value is observed at time t after insulation. In these circumstances the apparent resistance will be too low, owing to a part of the potential fall being due to causes other than leakage through the dielectric. It is not suggested that the cumbersome expression (21) be used to evaluate the resistance, but time should be allowed for the charge to become uniformly distributed before the higher potential reading is taken, after which the fall is simply exponential. In the case considered above an allowance of two seconds would be sufficient.

The foregoing reasoning would obviously apply equally well to the case of a long bar which is allowed to cool after one end has been maintained at a steady temperature for some time.

§ 6. ACKNOWLEDGMENT

The author's thanks are due to Mr G. E. Stevenson for suggesting the problem, and for his interest in its solution.

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(4) For example, E. Mallett, *Telegraphy and Telephony*, p. 34. (London, 1929.)
(5) *Loc. cit.* p. 184.

DISCUSSION

Capt. C. W. Hume. Does the theory take account of reflection at the receiving end?

J. H. Awbery. Mr McCleery states that the solution would answer also the question of the temperature-distribution in a long bar which is allowed to cool after one end has been maintained at a steady temperature for some time. Would he explain for what conditions this solution would apply? Is it when the bar is heat-insulated, or when its surface loses heat at a rate proportional to the excess above

air temperature? This question is equivalent to asking what are the boundary conditions in the electrical case actually solved—is the outside of the dielectric taken to be at the potential of the earth, and is this the same as the potential at the distant end of the cable?

I should like to ask also whether the expansion theorem as used here gives the same form of solution as would Fourier's method (though possibly with less labour), or does it give a different, and perhaps more rapidly convergent, series?

AUTHOR'S reply. In reply to Capt. Hume: reflection at the receiving end is taken care of by the tanh term in the expression for the current into the line.

Mr Awbery asks if the bar in the heat analogue is insulated. If this were the case the bar could not cool and the problem would not arise. Actually the lateral heat-loss from the bar, proportional to the excess above air temperature, corresponds to the leakage of electricity through the dielectric, proportional to the potential excess of the conductor above earth. The potential at the distant end of the cable is not zero unless the line is infinitely long; the theory assumes that the far end is free, i.e. insulated, and its potential at time $t = 0$ is given by

$$V_l = V_0 \operatorname{sech} l\sqrt{(RG)}.$$

In reply to the second part of the question: Fourier's method would require a large number of terms to approximate to the curve of potentials. In the present method the series is rapidly convergent; it was found for the numerical example quoted that the contribution from the fourth term of the series was quite negligible.

THE PROPAGATION ALONG THE EARTH OF RADIO WAVES ON A WAVE-LENGTH OF 1·6 METRES

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ABSTRACT. A previous paper described an investigation of the attenuation of ultra-short radio waves when transmitted directly along the earth's surface. The present paper reports the progress made in the continuation of this research, the particular wave-length to which attention has recently been given being 1·6 metres. A brief description is given of the simple, but efficient, transmitting and receiving apparatus which has been employed for the experiments on this wave-length.

Measurements of the field-intensity at different distances from the transmitter have been carried out for various heights of the apparatus above the ground level. When both transmitter and receiver are used very close to the ground, the attenuation curve obtained is similar to that encountered at longer wave-lengths. When, however, the apparatus is elevated by an amount comparable with, or greater than, the wave-length, the field-intensity-distance curves have maximum and minimum values, the positions of which depend upon the actual heights employed. These maxima and minima are due to interference between waves transmitted directly from the transmitter to the receiver, and those which arrive at the receiver after reflection from the earth's surface.

Theoretical curves having the same characteristics have been calculated from a consideration of the reflection coefficient of the earth's surface, account being taken of the electrical properties of the earth. By a comparison of such theoretical curves with the experimental results, the effective conductivity of the earth appears to be about 95×10^8 e.s.u. (resistivity 95 ohm-cm.) at the very high frequency of 190 megacycles per second employed. This is higher than the values of 5×10^8 to 30×10^8 e.s.u. previously obtained at frequencies of 30 to 60 megacycles per second, and these in turn were higher than the values obtained in earlier work, at a frequency of 1 megacycle per second. Owing to this considerable increase in the value of the conductivity as the frequency is raised, the experimental method does not enable the dielectric constant of the earth to be ascertained with any great accuracy, although a value of 10 gives suitable agreement between the theoretical and experimental results in the present case. At the same time this consideration indicates that, from the point of view of practical communications, the value of the dielectric constant of the earth is not a controlling factor in determining the propagation of waves over the earth's surface on either long or short wave-lengths, except in situations where the conductivity of the ground is abnormally low.

§1. SUMMARY OF PREVIOUS WORK

In a previous paper[1] the authors described an investigation of the attenuation of radio waves, of wave-length between 5 and 10 metres, when transmitted directly along the earth's surface. A comparison was made between the experimental results and those calculated from a simple wave-attenuation theory, and thereby values were found for the electrical conductivity of the earth for alternating-current

conditions at the frequencies of 30 to 60 megacycles per second employed. The present paper describes an attempt to extend this work in the direction of higher frequencies (i.e. shorter wave-lengths) with a view to increasing our knowledge both of the propagation of waves along the earth's surface and of the electrical constants of the earth. In particular it was desired, if possible, to obtain a more accurate knowledge of the effective dielectric constant of the earth, by operating at such a high frequency that this quantity might be expected to become as important as the conductivity in determining the mode of propagation of waves along the earth's surface.

In the region of frequencies now in question, some investigations have been made by M. J. O. Strutt[2] at a wave-length of 1·42 m. (210 Mc./sec.), and he has obtained a value of conductivity in excess of 10×10^8 e.s.u. for soil which at lower frequencies possessed a value of 1×10^8 e.s.u.

§2. DESCRIPTION OF APPARATUS AND METHODS EMPLOYED

(*a*) *Transmitter.* For the source of oscillations in these experiments, a low-power transmitter was constructed using a short-path-type receiving-valve connected to a single-tuned circuit with capacitative retroaction. The circuit arrangement adopted is shown in figure 1 *a*. The oscillatory circuit consisted of an inductance of a single turn, $1\frac{1}{2}$ in. in diameter, of $\frac{1}{8}$ in. copper tube, and a small variable air condenser having a maximum capacity of about 20 $\mu\mu$F. A similar condenser was employed for retroaction, while suitable choke coils, having a self-resonance at approximately the working wave-length, were used in the four direct-current supply leads to the valve electrodes. The valve, with its cap removed to reduce inter-electrode capacity, was mounted in an inverted position, and the oscillatory circuit was mounted as closely as possible to the leads from the valve electrodes. The tuning and retroaction condensers were controlled through extended handles from the front panel, while a filament resistance and grid-bias battery were mounted on the base-board. The direct-current supplies for the filament and anode circuit were obtained from batteries contained within a screened box upon which the set stood. The relative magnitude of the oscillatory current was measured by coupling to the inductance a single-turn loop connected to a non-contact-heater thermo-junction unit, the output side of which was connected to a pointer microammeter of suitable size for mounting on the set. This current-measuring combination was calibrated periodically with direct current, but as far as possible in each set of field-strength measurements the output from the transmitter was kept constant; a steady deflection on the microammeter was maintained by varying the voltage supplied to the valve anode circuit. Every effort was made to keep the dimensions of the apparatus as small as practicable in comparison with the working wave-length. In this connection the chief difficulty is encountered with the high-tension battery supply. In order to maintain a constant-current supply of the order of 20 mA. at 120 V. to the valve anode circuit, it was necessary to employ a large-size dry battery which, together with a 4-volt accumulator for the filament supply, was contained in a metal-lined box of

approximate dimensions 50 × 25 × 20 cm. The maximum dimension is thus seen to be less than one-third of a wave-length (160 cm.), and as, further, the choke coils employed were found to be very efficient in confining the oscillatory current to the tuned circuit, it is considered reasonable to assume that the loop inductance acts as a localized source of radiation for the purpose of this investigation.

(*b*) *Receiver*. The first stage of the receiver comprised a detector-oscillator unit which was identical with the transmitter just described. This stage was used in an oscillating condition for the heterodyne reception of the signals from the transmitter. After rectification the beat note of audible frequency was passed through a single amplifying stage to an output transformer connected to a pair of telephones. The audio-frequency potential-difference across these telephones was measured by means of a valve voltmeter of normal design and construction. A circuit diagram of the receiver is shown in figure 1 *b*.

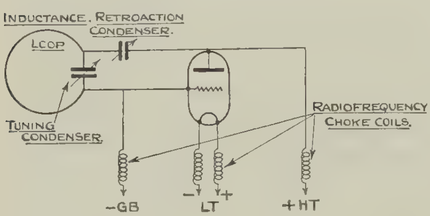

Fig. 1 (*a*).

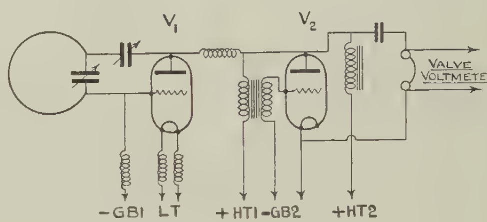

Fig. 1 (*b*).

An overall calibration of this receiving apparatus was carried out by measuring the output voltage for various values of the transmitter current to which the field-intensity at a fixed distance would be proportional. By this means it was ascertained that over a reasonably adequate working-range the output voltage was proportional to the field-intensity to an accuracy (of the order of 10 per cent) sufficient for the investigation.

(*c*) *Experimental procedure*. For the initial experiments the procedure was similar to that adopted in the case of the somewhat longer wave-lengths*. With the transmitter set in operation on a suitable site, measurements of the received field-intensity were made with the receiver at various distances from the transmitter while constant operating conditions were maintained throughout. The site employed for the measurements carried out at the National Physical Laboratory, Teddington, was the same as that used in the measurements on wave-lengths of from 5 to 10 m. as described in the former paper[1]. Owing to the shorter wave-length of 1·6 m. employed in the present case, however, it was necessary to take greater precautions to secure freedom from interference-effects from surrounding objects. The site chosen was flat and clear of all obstacles over an area of radius at least 100 m. from the transmitter, and as the majority of the measurements to be described were made at distances of less than 50 m., these conditions were considered to be satisfactory. On a wave-length of 1·6 m. great care has to be taken to ensure that the observers,

* See page 594 of reference (1), p. 508.

whose height averaged 1·8 m., did not produce spurious effects on the radiation. The first observer who controlled the transmitter was always at a reasonable distance from the apparatus, and during actual measurements he observed the deflection of the ammeter at the transmitter by the aid of a telescope. At the receiving end the second observer was always placed in a position below the set, which in some cases involved lying flat on the ground. In any series of measurements the positions of both observers were maintained constant in relation to the apparatus.

In order to study the propagation of waves directly along the earth's surface, it is necessary that the heights above the ground of the radiating and receiving points should be small compared with the wave-length. A practical limit was set in this direction by the dimensions of the apparatus. With the transmitter standing on its battery box placed on the ground, the height of the small radiating loop above the earth's surface was 50 cm. The height of the receiving loop similarly placed was 53 cm. For other measurements the heights of the transmitter and receiver were increased up to about 3 m. from the ground by placing the apparatus on portable wooden tables or stools.

§ 3. DISCUSSION OF RESULTS OBTAINED

(*a*) *Experimental results.* When the heights of the transmitting and receiving loops were about 0·50 m. or less than one third of the wave-length, it was found that the field-intensity decreased steadily as the distance was increased, so that the

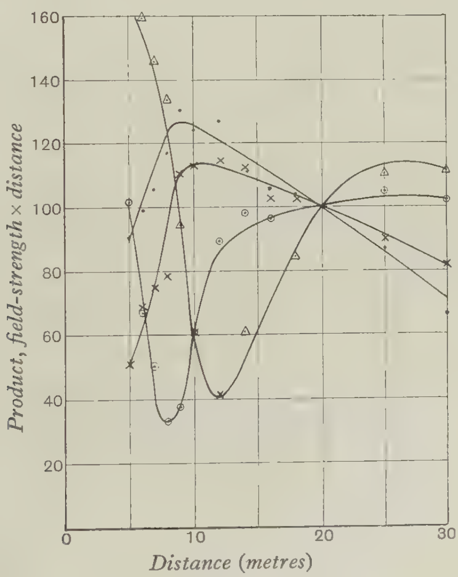

Fig. 2. Experimental results. Wave-length, 1·6 m.; height of receiver, 1·4 m.; height of transmitter, 0·5 m. (dots), 1·2 m. (crosses), 2·1 m. (circles), 2·8 m. (triangles).

attenuation curve was of the type normally experienced on longer wave-lengths. When, however, the transmitter and receiver were raised to a height comparable

with or greater than the wave-length, the field-intensity passed through successive maximum and minimum values as the distance between transmitter and receiver was increased. Figure 2, for example, shows a set of experimental results plotted for a fixed height of receiver and for four different heights of the transmitter. These graphs show, in the usual manner, the relation between the distance and the product of field-intensity and distance. It is seen that this product passes through maximum and minimum values, the position of which depends upon the height of the transmitter. As the transmitter is raised the maximum moves out from its first position at a distance of about 9 m. to its last position at a distance of 27 m. This maximum is followed by a trough or minimum which is at a distance of less than 5 m. for the first two heights and then occurs at distances of about 8 and 12 m. for transmitter heights of 2·1 and 2·8 m. These curves represent, in fact, a type of interference pattern in the radiated field, the positions of the fringes depending upon the height of the transmitter above the ground.

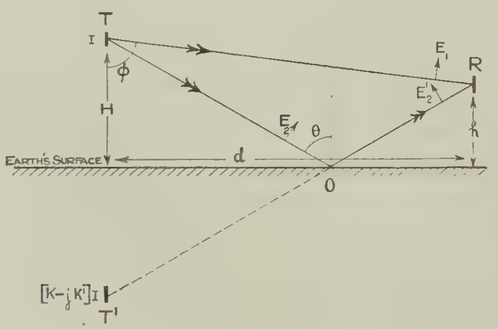

Fig. 3.

The mode of production of this interference pattern can easily be understood by the aid of figure 3. This diagram represents the transmitter T and receiver R, at heights H and h above the earth's surface. It is evident that two sets of waves will reach R from T; one along the direct path TR, and the other along the path TOR, after reflection from the ground at the point O. The magnitude of the resulting field at R will depend upon the amplitudes of these two waves and also upon their phase-difference. The phase-difference will in turn depend upon the actual path-difference of the waves and on the phase-change introduced on reflection at the ground. The intensity of the resulting field at the receiver passes through maximum or minimum values according as the two sets of waves are approximately in the same or in opposite phase.

The direction of the electric field in a plane wave is normal to the direction of propagation of the wave, so that the two fields arriving at R may be represented by the two vectors E_1, E_2'. If an aerial were used for reception at R the difference in direction between E_1 and E_2' would have to be taken into account when the field-intensity at R is calculated theoretically. A small loop at R, however, has the same reception characteristics for all directions in the plane of the coil. Similar conditions

apply at the transmitter T, so that the intensity of radiation from T along the two paths TR and TO are the same. The ray TOR, however, undergoes reflection at O, the coefficient of reflection being determined by the boundary conditions at O in accordance with Fresnel's equations. These equations are strictly valid only if the distances of transmitter and receiver from the reflecting surface are such that the wave incident at the surface is plane and also that the re-radiation from the surface becomes plane before it reaches the receiver. These two conditions were fulfilled in the majority of the experiments by keeping the transmitter and receiver at minimum distances of about one wave-length above the ground. Even in the case in which the heights of transmitter and receiver above the ground were only 0·5 m. (less than one-third of the wave-length), Fresnel's equations appeared to apply to an approximate extent; but it is proposed to examine this point in more detail later.

(*b*) *Theoretical curves.* Suppose that the complex coefficient of reflection of the ray TO incident at the angle θ at O, figure 3, is represented by $(K-jK')$ in which θ, K, K'
j is equal to $\sqrt{-1}$. It can be shown from Fresnel's equations that j

$$K-jK' = \frac{(k^2+4\sigma^2/f^2)\cos^2\theta-(c^2+d^2)-2j\cos\theta\,(kd+2c\sigma/f)}{(k^2+4\sigma^2/f^2)\cos^2\theta+(c^2+d^2)+2\cos\theta\,(kc-2d\sigma/f)},$$

where σ is the conductivity and k is the dielectric constant of the ground, f is the frequency of the wave incident on the earth's surface at the angle θ, and the values of c and d are given by σ, k, f c, d

$$c^2-d^2=k-\sin^2\theta,$$

$$cd=-\sigma/f.$$

The values of K and K' can be calculated for all angles of incidence, θ, and a selection of graphs for some typical values of σ and k were given in a paper published by Wilmotte (3).

The problem of calculating the field-intensity at R is simplified if we consider the radiation along the path TR to come from a source at T of strength, say, I, and the radiation along TOR to come from a similar source at T' of strength $(K-jK')\,I$, where T' is the image of T in the earth's surface. The field at R due to I is proportional to I/TR, and that due to the source $(K-jK')\,I$ is proportional to I

$$(K-jK')\,I/T'R,$$

and these two fields differ in phase by $(2\pi/\lambda)\,(T'R-TR)$. It can be shown that the product of the resultant field at R and the horizontal distance between T and R is proportional to

$$\Big[\sin^2\phi+K^2\sin^2\theta+K'^2\sin^2\theta+2K\sin\theta.\sin\phi.\cos\frac{2\pi}{\lambda}(T'R-TR)$$
$$-2K'\sin\theta.\sin\phi.\sin\frac{2\pi}{\lambda}.(T'R-TR)\Big]^{\frac{1}{2}},$$

where ϕ is the angle the direction TR makes with the vertical.

A set of theoretical curves calculated from this formula for fixed heights of transmitter and receiver and for various typical values of k and σ/f is given in figure 4.

These curves show, as before, the relation between the product of field-intensity times distance, and for distances up to 30 m. from the transmitter. By inspection of these curves it can be seen immediately that the dip in the curve corresponding to a minimum value of the product of signal-strength and distance becomes more marked as the value of σ/f is raised from 5 to 100. An increase in the value of k from 0 to 10 is also effective in reducing the depth of the trough although to a less marked extent. An additional effect of increasing the value of k is that it moves the position of the minimum towards the transmitter.

(*c*) *Comparison of theory and experiment.* By superimposing the three most suitable curves with the corresponding experimental results we obtain figure 5 for a transmitter-height of 2·1 m. From this comparison it is seen that the most appropriate value of σ/f is about 50, with k equal to 10. It is considered that such values serve to locate the position of the trough on both coordinates to within the probable experimental accuracy. Similar comparisons have been carried out for two other heights of the transmitter, and the results are reproduced in figures 6 and 7. In all the cases so far investigated it appears that the theoretical curves calculated for $\sigma/f = 50$ and $k = 10$ fit the experimental results to within the accuracy (about 10 per cent) that is at present obtainable in working on such short wave-lengths. It must be pointed out that with the large value of σ/f which appears in this result, the method does not enable the value of k to be determined with any great accuracy, since within the limits 0 and 10 the value of k does not greatly affect the actual location of the curves.

In this respect one of the objects of carrying out this investigation has failed, for if σ is given the value of 10^8, which is found for wave-lengths of the order of 300 metres (frequency 1000 kc./sec.), the value of σ/f at a wave-length of 1·6 m. (frequency 190 Mc./sec.) would be about 0·53 and it is likely that k would have been the controlling factor in determining the coefficient of reflection of the waves. This conclusion indicates that, from a practical point of view, the value of the dielectric constant k is not a controlling factor in determining the propagation of radio waves over the earth's surface on either long or short wave-lengths, except in situations where the conductivity of the ground is abnormally low.

(*d*) *Value of conductivity of the earth.* From the results of the investigation described in the previous paper[1], it was seen that the value of the effective conductivity of the earth determined for wave-lengths between 5 and 10 m. (frequencies 60 to 30 Mc./sec.) was within the limits 5×10^8 and 32×10^8 e.s.u., whereas the values previously obtained on wave-lengths between 350 and 750 m. (frequencies between 860 and 400 kc./sec.) lay between $0{\cdot}2 \times 10^8$ and 4×10^8 e.s.u. The results of the experiments described in the present paper indicate that on the wave-length of 1·6 m. (frequency 190 Mc./sec.) the conductivity of the earth in the same locality is about 95×10^8 e.s.u.* This affords some confirmation of the fact that there appears to be a definite and substantial increase in the effective value of the earth's conductivity under alternating-current conditions as the frequency is increased above one

* The corresponding resistivity is 95 ohm-cm.

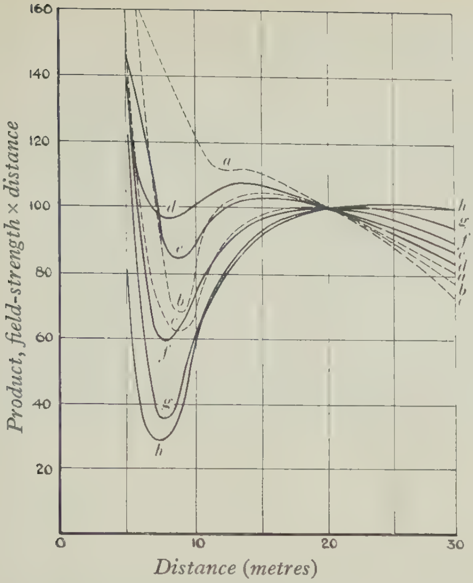

Fig. 4. Theoretical results.

$a : \kappa=0\ \sigma/f=5$	$d : \kappa=10\ \sigma/f=5$	Height of transmitter = 2·1 m.
$b : \kappa=0\ \sigma/f=10$	$e : \kappa=10\ \sigma/f=10$	
$c : \kappa=0\ \sigma/f=20$	$f : \kappa=10\ \sigma/f=20$	
	$g : \kappa=10\ \sigma/f=50$	Height of receiver = 1·4 m.
	$h : \kappa=10\ \sigma/f=100$	

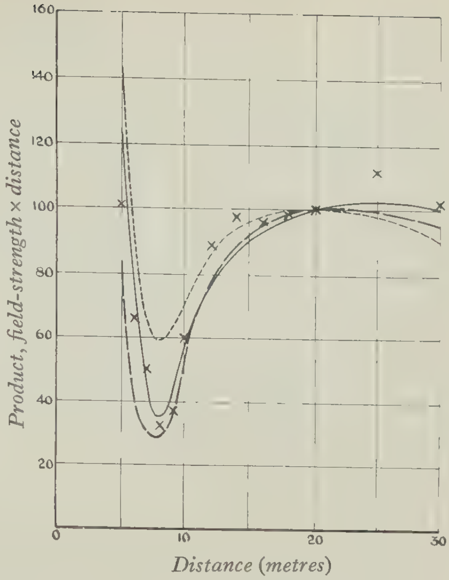

Fig. 5. Comparison of theoretical and experimental results.

Height of transmitter = 2·1 m. }
Height of receiver = 1·4 m. } $\lambda = 1{\cdot}6$ m.
Calculated curves: ---- $\kappa=10 : \sigma/f=20$
—— $\kappa=10 : \sigma/f=50$
— — $\kappa=10 : \sigma/f=100$
Experimental results: ×

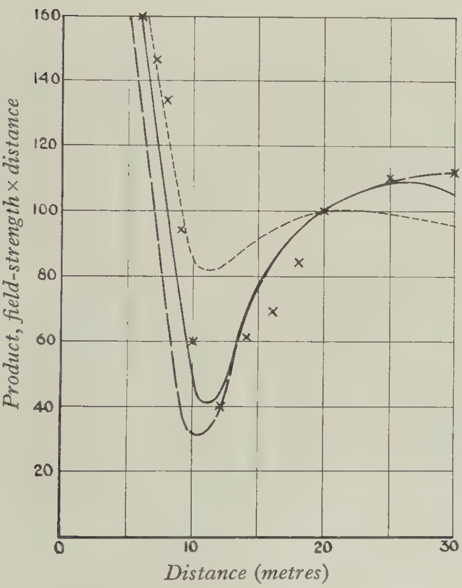

Fig. 6. Comparison of theoretical and experimental results.

Height of transmitter = 2·8 m. }
Height of receiver = 1·4 m. } $\lambda = 1{\cdot}6$ m.
Calculated curves: ---- $\kappa=10 : \sigma/f=20$
—— $\kappa=10 : \sigma/f=50$
— — $\kappa=10 : \sigma/f=100$
Experimental results: ×

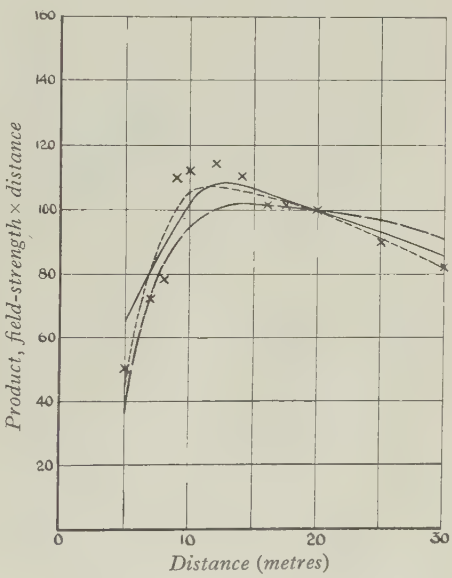

Fig. 7. Comparison of theoretical and experimental results.

Height of transmitter = 1·2 m. }
Height of receiver = 1·4 m. } $\lambda = 1{\cdot}6$ m.
Calculated curves: ---- $\kappa=10 : \sigma/f=20$
—— $\kappa=10 : \sigma/f=50$
— — $\kappa=10 : \sigma/f=100$
Experimental results: ×

million cycles per second. This view, however, is put forward somewhat tentatively as the authors realize that in the field experiments which they have conducted the experimental accuracy tends to decrease as the frequency rises. It is proposed to continue the experiments, as opportunity offers, over the range of wave-lengths from 1 to 10 m. in an endeavour to obtain greater accuracy. In the continuation of the investigation careful attention would be paid to ascertaining the exact position of the effective reflecting surface. This may not always be at the ground level and may vary with the moisture-content of the ground, chiefly when the surface conditions are very dry. Since a depth of soil of an appreciable fraction of a wave-length may be involved in the reflection, this point can be most suitably investigated at very short wave-lengths.

§4. ACKNOWLEDGMENTS

The work described in this paper was carried out as part of the programme of the Radio Research Board, and acknowledgment is due to the Department of Scientific and Industrial Research for granting permission for publication. The authors desire to thank Messrs A. C. Haxton and H. M. Bristow for much assistance both in the experimental measurements and in connection with the various theoretical calculations involved.

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DISCUSSION

Dr D. Owen. It is interesting to observe in these short-wave experiments the existence of maxima and minima of intensity as the receiver is carried further back. The measurements thus provide an important means of studying reflection from the ground. There must, however, be considerable uncertainty as to the point O to be chosen in using the diagram of figure 3 for the purpose of applying the Fresnel formula. It should be of material assistance therefore to obtain data when a perfectly reflecting surface is used, and it seems feasible, at such short wave-lengths, to suggest the use of a plane metallic reflector. A long strip, say 5 to 10 m. long by 2 m. wide, of the finest copper sheet might be mounted on a light frame for this purpose. If suitably inclined it would also serve to divert the reflected beam completely from the receiver, so permitting the intensity of the direct beam to be separately ascertained.

In regard to the numerical values, I should like to ask to what extent the variations between the limits quoted depend on wave-length, and how far on the nature and degree of wetness of the soil.

Mr F. D. Smith. The equation in § 3 (*b*) reduces to

$$K - jK' = \frac{(2\sigma/f)\cos^2\theta - 1 + 2j\,(\sigma/f)^{\frac{1}{2}}\cos\theta}{(2\sigma/f)\cos^2\theta + 1 + 2\,(\sigma/f)^{\frac{1}{2}}\cos\theta}$$

when $$\sin^2\theta \ll k \ll \sigma/f,$$

differing in the sign of the imaginary term from the corresponding equation (29) of R. M. Wilmotte's paper*. Now the experimental results indicate values

$$k = 10, \quad \sigma/f = 50,$$

so that this simplified formula is applicable in the present case. In the extreme case

$$(2\sigma/f)\cos^2\theta \gg 1, \quad K \fallingdotseq 1 \quad \text{and} \quad K' \fallingdotseq 0,$$

and in the other extreme case

$$(2\sigma/f)\cos^2\theta \ll 1, \quad K \fallingdotseq -1 \quad \text{and} \quad K' \fallingdotseq 0.$$

The reflection is practically perfect except when $(2\sigma/f)\cos^2\theta$ does not differ greatly from unity, that is, except when $\cos^2\theta$ does not differ greatly from 0·01. When the reflection is practically perfect, the interference pattern is practically independent of σ/f and no estimate of the numerical values of σ/f can be made. On the other hand, when $\cos\theta \fallingdotseq 0{\cdot}1$ the most favourable condition for forming an estimate of σ/f obtains. In these circumstances would it not have been better to have equalized the theoretical curves and the experimental results at a shorter distance of, say, 5 m., where the reflection is practically perfect?

The general agreement between theory and experiment, which is particularly good in view of the experimental difficulties encountered at 1·6 m., indicates the valuable conclusion that the earth behaves as a highly conducting sheet practically impenetrable to radio waves of wave-length 1·6 m. Similar experiments with shorter waves may give very complex results. The boundary between the two media is somewhat indefinite and a region of gradual transition over several wave-lengths may sometimes exist. In these circumstances, refraction and reflection may both take place.

Authors' reply. In reply to Dr Owen: the uncertainty in our knowledge of the exact point of reflection from the ground was realized by us, and a brief reference to the further experiments to be carried out in this connexion is made at the end of § 3 (*d*) of the paper. We had also appreciated the desirability of using metallic reflecting sheets laid over the ground surface and we have actually commenced experiments on these lines. In the first place, for reasons of economy, we are using small-mesh galvanized iron wire netting and have acquired a piece of dimensions 16 m. × 4 m., which should be adequate as a reflecting surface for a wave-length of

* *Loc. cit.*

1·6 m. With the aid of this netting we shall endeavour to determine the smallest size of sheet actually required, and we shall then replace this with a sheet of solid copper or copper gauze. We also visualize the possibility of extending the experiments to reflection from a sheet of water, in particular with a view to measuring the electrical properties of sea-water at high radio frequencies. The variation in the values of the conductivity of the soil with wave-frequency given in the paper are quite definite and are much larger than the variation experienced with the moisture-content of the soil under normal field conditions in this country. A laboratory investigation of the electrical properties of soil at various frequencies and over a wide range of moisture-contents is also in progress, however, and the results of this should provide more exact knowledge on these points.

In reply to Mr F. D. Smith; the error in sign in the formulae in Wilmotte's paper to which he refers was noticed some time ago, and the corrected form was actually used by us in our previous paper published last year. It appears to us that in reducing this formula by making various approximations for extreme cases, Mr Smith is elucidating the form of the graphs for K and K' (given in figures 4 and 5 of R. M. Wilmotte's paper*) for these cases. Incidentally the fact that for the case when $(2\sigma/f)\cos^2\theta \ll 1$, K is equal to -1 and not $+1$ gives rise to a certain difficulty, since the direct and reflected waves are $180°$ out of phase with each other, and the total field-strength at the surface would appear to be zero, a deduction which is not borne out by experiment. The explanation of this anomaly is that, as explained in the paper, the equations from which the formula for the reflection coefficient was derived do not hold for the case in which the receiver (or transmitter) is very close to the surface. With regard to the choice of distance at which the equalizing of the curves and results was carried out, 20 m. was selected because it was considered that at this distance the overall accuracy of the experiments was greatest.

We agree with Mr Smith's remarks on the boundary conditions which may lead to complex results at shorter wave-lengths, and our future investigations will be directed towards obtaining more knowledge on this point. It may be stated, however, that part of the object of working in the wave-length-range 1 to 10 m. was to investigate this complexity, due to the supposed increasing importance of k at very high frequencies. As our results show, however, the value of σ has increased so much at these very short wave-lengths that the value of $2\sigma/fk$ is still large and thus k appears to play only a minor part in the properties of soil at high frequencies.

* *Loc. cit.*

NOTES ON SURFACE-TENSION MEASUREMENT

By ALLAN FERGUSON, M.A., D.Sc., F.Inst.P. and
S. J. KENNEDY, B.Sc.

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ABSTRACT. The first part of this paper deals with a method for the accurate determination of the surface tension of liquids available in volumes of not more than one or two cubic millimetres. The method described does not involve any knowledge of the density of the liquid.

The second part of the paper describes a series of measurements of the variation with concentration of the surface tension of aqueous solutions of *p*-toluidine. Here, also, the method employed is independent of a knowledge of the density of the solution.

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§ 1. THE MEASUREMENT OF SURFACE TENSION USING A SMALL QUANTITY OF LIQUID

SOME time ago, one of us (A. F.)* pointed out that it was possible to obtain accurate values for air-liquid or interfacial tensions using as little as one or two cubic millimetres of liquid. The method then described consisted in drawing a short thread of liquid into a vertical capillary tube 1 mm. or less in bore, attaching the capillary to a manometer and to a simple arrangement for varying the pressure, and increasing the pressure to a value $g\rho_1 h_1$ such that the thread of liquid g, ρ_1, h_1
in the capillary is forced down until the surface of the liquid at the lower end of the capillary is *plane*. If ρ_2 is the density of the liquid under test, and h_2 the length of the ρ_2, h_2
thread of liquid, we than have

$$\gamma = \tfrac{1}{2} g R\,(\rho_1 h_1 + \rho_2 h_2) \qquad \text{......(1)},$$

exactly, where R is the radius of curvature of the upper meniscus at its vertex, and R
γ the surface tension of the liquid. Assuming a zero contact angle and substituting γ
for R the value

$$R = r\,(1 + r^2/6a^2) \qquad \text{......(2)},$$

where r is the internal radius of the tube and as usual $a^2 \equiv \gamma/g\rho_2$, we find r, a

$$\gamma = \tfrac{1}{2} g r\,(\rho_1 h_1 + \rho_2 h_2) + \tfrac{1}{6} g \rho_2 r^2 \qquad \text{......(3)},$$

as the working equation from which to determine the surface tension.

Obviously, the equation as it stands necessitates a knowledge of ρ_2, the density of the liquid under test, and this determination may be a matter of some difficulty if only a few cubic millimetres of liquid are available. Two courses are open; we may, without appreciable loss of accuracy, shorten the length of the thread of liquid until $\rho_2 h_2$ is negligible in comparison with $\rho_1 h_1$, or we may make a number of

* *Proc. Phys. Soc.* **36**, 37 (1923).

determinations, varying the value of h_2 and, writing the equation for γ in the form

$$h_1 = -\frac{\rho_2}{\rho_1}\left(h_2 + \frac{r}{3}\right) + \frac{2\gamma}{gr\rho_2} \qquad \ldots\ldots(4),$$

we see that if we plot h_1 against $(h_2 + \frac{1}{3}r)$ we can obtain γ without the necessity for a direct determination of ρ_2.

In this paper we consider another method of obviating this difficulty—that of using the capillary tube in a horizontal position. This not only relieves us from the necessity of determining the density of the liquid under test, but simplifies a little the experimental arrangements. Figure 1 shows the disposition of the apparatus*.

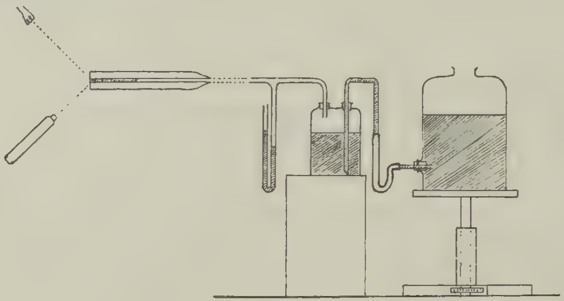


Fig. 1.

By means of the simple arrangement shown on the right-hand side of the drawing, the liquid thread is forced along the capillary until the meniscus at the open end of the tube, at first concave, flattens out until it is exactly plane. Further increase of pressure makes the meniscus convex. A number of experimenters who have used the method previously described seem to have found difficulty in hitting off the plane-position. We therefore show in the plate, *b–f*, a series of photographs of the end of the capillary: *b* and *c* show concave menisci, *e* and *f* convex menisci, and *d* as nearly as possible the plane position. The corresponding pressures, as read on an aniline manometer, are shown in table 1.

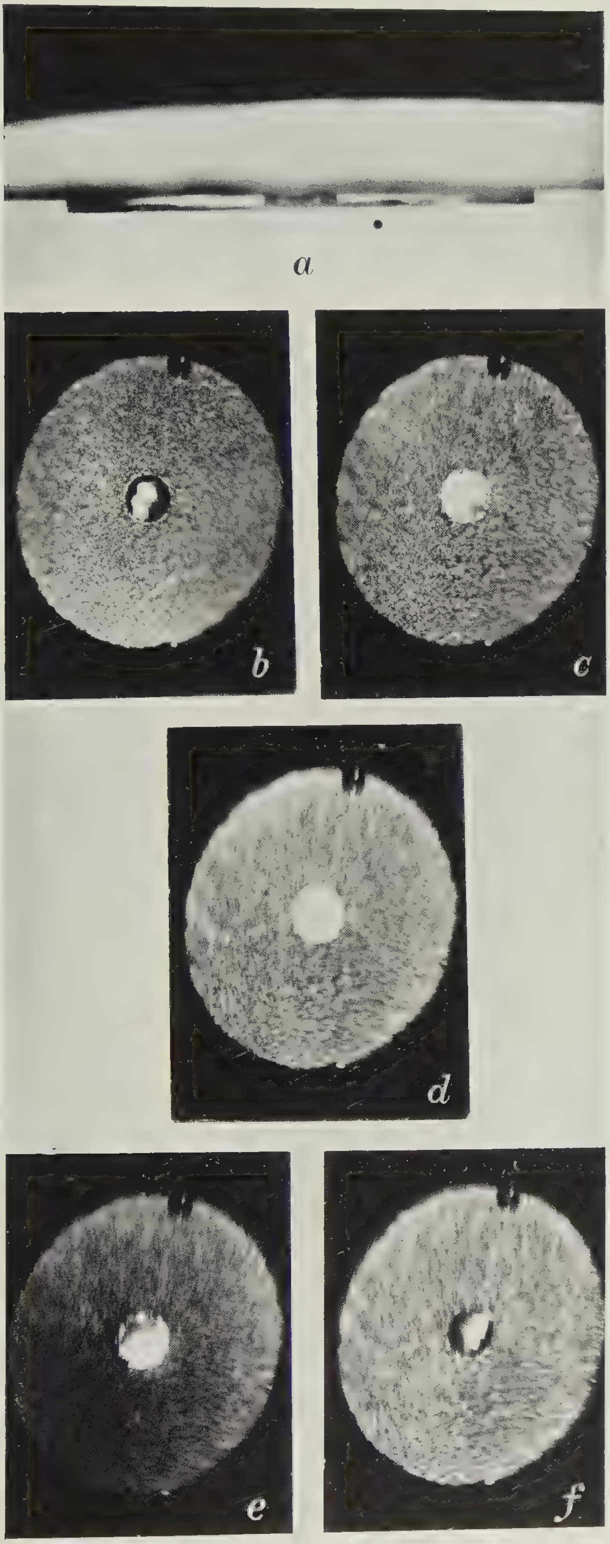
Table 1.

Photograph	*b*	*c*	*d*	*e*	*f*
Pressure (cm. aniline)	2·689	2·836	2·887	2·955	3·108

It must not be imagined that these steps of pressure in any way represent the delicacy of the setting, which is very sensitive and clear-cut. The steps are made at intervals as large as 0·5 mm. or thereabouts simply to show more clearly on the photographs the march of the observed phenomena.

Obviously, the bore of the tube used must be small enough to ensure that there is no serious gravitational distortion of the meniscus. The problem of the distortion

* It is obvious in the figure that the drawing is not all made to the same scale. The horizontal capillary on the left is much enlarged for convenience. Moreover, the filament lamp and low-power microscope for viewing the image of the filament in the meniscus are, of course, situated in a horizontal plane perpendicular to the plane of the rest of the drawing.



[*To face page* 512

of the meniscus in a horizontal capillary tube is not a promising subject for mathematical attack, and, fortunately for our purpose, it can be discussed on a purely experimental basis. We have seen that if a column of liquid of length h_2 requires a head h_1 of another liquid to force it to the "plane" position, then

$$2\gamma/gr = \rho_1 h_1 + \rho_2 (h_2 + \tfrac{1}{3}r) \qquad \text{......(5).}$$

If we have a horizontal tube and find that a manometer liquid of density ρ_1 and giving a head h_1' forces the liquid under test into the plane position, we may put

$$2\gamma/gr = \rho_1 h_1' \qquad \text{......(6),}$$

if the tube is sufficiently narrow to enable us to neglect gravitational forces in comparison with those due to surface tension.

The procedure adopted was to make a test first with the tube vertical, then with the tube horizontal, and to compare the values of $\rho_1 h_1 + \rho_2 (h_2 + \frac{1}{3}r)$ and of $\rho_1 h_1'$. If this test is carried out with tubes of different bore, it is not difficult to find an outside limit for the bore of a tube which may be safely employed with liquids whose surface tensions and densities are of the order usually encountered. Thus using aniline as the test liquid and manometer liquid, and a tube of just less than a millimetre bore, all that is necessary is to compare the values of h_1' and $h_1 + h_2 + \frac{1}{3}r$. The figures in table 2 were obtained.

Table 2.

Radius of capillary, 0·045 cm. Temperature, 16·5° C.
Test liquid, aniline. Manometer liquid, aniline.

h_1' (cm.)	1·936	1·938	1·937	1·933	1·933	
$h_1 + h_2 + \frac{1}{3}r$ (cm.)	1·938	1·938	1·938	1·935	1·934	1·935

Mean value of $h_1 + h_2 + \frac{1}{3}r$ = 1·936 cm.
Mean value of h_1' = 1·934 cm.

If the test liquid differs in density from the manometer liquid, then the comparison must be made as mentioned above. Such a comparison has been made using water as the test liquid, aniline as the manometer liquid, and tubes of three different radii. The results corrected to read in centimetres of aniline are shown in table 3.

Table 3.

Internal diameter of tube (cm.)	h_1' (cm.)	$h_1 + h_2 + \frac{1}{3}r$ (cm.)
0·126	2·410	2·407
	2·379	2·389
	2·422	2·413
	2·363	2·362
0·100	3·064	3·054
	3·145	3·139
	3·051	3·058
	3·023	3·031
0·092	3·213	3·225
	3·227	3·218
	3·213	3·202
	3·176	3·167

It seems, then, that with tubes of less than 1 mm. bore, we are quite safe in employing a horizontal tube and in using the simple formula

$$\gamma = \tfrac{1}{2} rh\rho g \qquad \ldots\ldots(7),$$

where h and ρ refer, of course, to the manometer liquid. An experiment was carried out to determine the surface tension of water, and the simple formula just given was used. The results are given in table 4.

Table 4.

Tube 1. Radius 0·036 cm. Temperature 15° C.

h (cm.)	4·070	4·060	4·069	4·069	4·064	4·068

Mean value of h = 4·067 cm.

γ = 73·47 dyne/cm.

Tube 2. Radius 0·046 cm. Temperature 15° C.

h (cm.)	3·188	3·184	3·189	3·181	3·182	3·191

Mean value of h = 3·186 cm.

γ = 73·55 dyne/cm.

Tube 3. Radius 0·063 cm. Temperature 15° C.

h (cm.)	2·320	2·323	2·317	2·322	2·320	2·321

Mean value of h = 2·320 cm.

γ = 73·38 dyne/cm.

Surface tension of water γ = 73·47 dyne/cm. at 15° C.

Finally, an attempt was made to determine a number of interfacial tensions. Suppose we have two liquids α and β in contact in a horizontal capillary tube. If the pressure inside the tube is increased to an excess value $g\rho h$ sufficient to make plane the surface of the liquid β at the open end of the capillary tube, a simple argument shows that, if we assume zero contact angles throughout,

$$\gamma_{\alpha\beta} = -\gamma_\alpha + \tfrac{1}{2} rh\rho g \qquad \ldots\ldots(8).$$

The results of experiments made on six different liquids are shown in table 5.

Table 5.

Liquid	γ_α (dyne/cm.)	$\gamma_{\alpha\beta}$ (dyne/cm.)	Temperature (° C.)
Ether	17·41	10·56	16·0
Carbon tetrachloride	27·05	44·98	16·5
Benzene	29·14	33·67	15·0
Chloroform	27·50	30·02	15·0
Toluene	28·84	37·73	16·0
Ethyl bromide	24·52	31·16	17·0

The interfacial tensions are all determined against water, and the values are in very fair agreement with those determined by different methods. The values for γ_α cited in table 5 were determined by this method.

To sum up: whilst the method, in common with most capillary tube methods, assumed a knowledge of the contact angle, in most cases assumed to be zero, as at present described it may fairly be called accurate and has many advantages over the ordinary capillary-rise method. (1) The technique is simpler and cleaning operations are more easily carried out. (2) No knowledge of the density of the liquid under test is required. (3) The amount of liquid used may be cut down to one or two cubic millimetres, without any loss of precision. (4) For tubes of less than a millimetre bore, the simple formula may be applied. Hence it follows that the manometer *may be made direct-reading, as its indications need only be multiplied by a factor which, for a given capillary, is constant.* There is, of course, no restriction to the U-tube type of manometer. It is conceivable that a robust and yet sensitive direct-reading instrument may be constructed employing a manometer of aneroid type.

§ 2. ON THE SURFACE-TENSION/CONCENTRATION CURVES FOR AQUEOUS SOLUTIONS OF *p*-TOLUIDINE

Some time ago, a method for surface-tension measurements was described by one of us (A. F.) in a paper published in collaboration with Mr P. E. Dowson*. The method consisted in immersing a capillary vertically to a depth h_2 in the liquid of density ρ_2 under test, and measuring on a separate manometer the pressure $g\rho_1 h_1$ required to force the liquid meniscus to the bottom of the capillary and to hold it there.

The method has obvious advantages over the capillary-rise method inasmuch as: (1) No calibration of the tube is necessary. (2) Pressure measurements are made on a separate manometer and may be magnified at will. (3) The cleaning of the apparatus is facilitated. (4) There is very little adsorption at the glass-liquid surface of the tube, as compared with the amount likely to take place in measurements made by the ordinary capillary-rise method; this is a matter of some importance in measurements made on solutions.

The method described in the first part of this paper, the capillary-rise method, and the method used in the measurements now to be described, are all closely allied. In each instance a *pressure excess* and a *curvature* are measured; in each instance a *plane* surface of reference is provided.

In the capillary-rise method the liquid under test acts as its own manometer, and the pressure required is determined by measuring the vertical distance between the meniscus vertex and the plane surface of the liquid in the outer container. In the method described in the first part of this paper the *total* pressure excess is measured on a separate manometer, and the plane surface of reference is obtained as described. In the present method the pressure excess required is, small corrections apart, measured as the *difference* between the pressure read on the manometer and the pressure due to the immersion of the meniscus to a depth h_2 below the *plane* surface of the liquid under test.

* *Manchester Memoirs*, **65**, No. 5 (1921); *Trans. Far. Soc.* **17**, 384 (1921).

A simple argument shows that, for a liquid of zero contact angle, the surface tension is given by

$$\gamma = \tfrac{1}{2}gr\,(\rho_1 h_1 - \rho_2 h_2) + \tfrac{1}{6}g\rho_2 r^2 \qquad \text{......(9)}.$$

If we vary h_2 and measure the corresponding values of h_1 we see that, rearranging equation (9) above, we may put

$$h_1 = \frac{\rho_2}{\rho_1}\left(h_2 - \frac{r}{3}\right) + \frac{2\gamma}{g\rho_1 r} \qquad \text{......(10)}.$$

Hence, if we plot h_1 as ordinate against $(h_2 - \frac{1}{3}r)$ as abscissa, the intercept of the line so obtained gives the surface tension of the liquid and obviates any necessity for the determination of its density.

But we may avoid the density-determination in another manner. If, as a particular case, we make h_2 equal to $\frac{1}{3}r$, we have

$$\gamma = \tfrac{1}{2}g\rho_1 h_1 r \qquad \text{......(11)}$$

as a very simple formula for the evaluation of γ, necessitating no knowledge of ρ_2 and yet taking into account the small correcting term of the first order. The uppermost photograph in the plate shows that it is quite easy, with a tube having a bore of a millimetre or less, to immerse the tube to the required depth and to observe accurately the equilibrium position of the meniscus at the lower end of the tube. The adjustment is easily made, either by raising the vessel containing the liquid under test until, as shown by observation of the image of the bottom of the tube reflected in the liquid surface, the end of the tube is just touching that surface, and then racking up the vessel through the additional third of the radius; or with the assistance of a needle point fixed to the capillary at a known vertical distance from the end of the tube, and far enough away from the tube to ensure its touching a portion of the liquid surface that is sensibly plane. In either case, it must be remembered
a that if a thick-walled capillary tube of external cross-section *a* is employed, when it has been racked down into the liquid through a distance equal to $\frac{1}{3}r$, the immersion may not be equal to this on account of the liquid displaced by the tube. A correcting
A factor depending on a/A, where A is the cross-sectional area of the vessel containing the liquid, is necessary, and this correction must be made or precautions must be taken to ensure that it is negligible.

Observations of the variation of the surface tension of aqueous solutions of *p*-toluidine were made some years ago by Edwards*, using the method as described by Ferguson and Dowson†. Later, Gans and Harkins‡ covered the same ground using a drop-weight method, and obtained a curve between surface tensions and concentration, differing considerably from the curve obtained by Edwards. The matter was investigated further by R. C. Brown§, who determined the concentration/surface-tension relations by three or four different methods and found that the values obtained from Ferguson's method did not disagree with the results of the

* *J. Chem. Soc.* **127**, 744 (1925).
† *Trans. Far. Soc.* **17**, 384 (1921).
‡ *J. Am. Chem. Soc.* **52**, 2289 (1930).
§ *Phil. Mag.* **11**, 686 (1931).

experiments of Gans and Harkins. It seemed, therefore, that a useful test of the technique just described would be found in a series of measurements of the surface tension of aqueous solutions of *p*-toluidine.

Para-toluidine of research quality was obtained and was recrystallized from aqueous alcohol. The crystals so obtained were dried in a desiccator for several days.

Aqueous solutions of known concentration—always expressed as x gm. in 100 cm³ of water—were made up, and the surface tensions were measured in the manner indicated*.

The radius of the tube was measured by means of a travelling microscope made by Watson and reading to 0·01 mm., and the pressures, measured by means of a simple U-tube manometer containing aniline, were read on the same instrument. The usual precautions in respect of cleanliness were taken. The results are shown in table 6.

Table 6. Authors' results for *p*-toluidine at 16° C.

Concentration (gm./100 cm³)	Pressure (cm. aniline)	γ (dyne/cm.)
0·0	3·167	73·10
0·0233	3·082	71·11
0·0467	3·082	71·11
0·0935	3·030	69·94
0·100	3·031	69·96
0·131	2·953	68·16
0·150	2·933	67·70
0·187	2·846	65·69
0·200	2·811	64·88
0·225	2·727	62·95
0·250	2·679	62·30
0·262	2·644	61·00
0·300	2·549	58·83
0·333	2·510	57·99
0·375	2·462	56·82
0·450	2·310	53·32
0·500	2·257	52·01
0·600	2·140	49·36

Table 7. Gans and Harkins's results for *p*-toluidine at 20° C.

Concentration (gm./100 cm³)	γ (dyne/cm.)
0·0	72·75
0·0649	71·63
0·0767	71·38
0·1257	69·82
0·2126	64·57
0·2944	60·38
0·3616	57·73
0·5286	52·35
0·6659	49·10

* In every case, tap water was employed for the manufacture of the solutions.

The agreement with the observations of Gans and Harkins, which were made at a temperature of 20° C., is best shown by means of the curve of figure 2. Here the full line represents the results of our own experiment—the actual figures obtained are shown thus ⊙—and the small crosses show the numbers given by Gans and Harkins. To make the comparison strict, since our own figures were obtained at 16° C., the full line should be moved slightly downwards—how much it is difficult to say, as the temperature-coefficient is unknown—but it is not likely that the shift will be greater than 0·4 or 0·5 of a unit of surface tension. The dotted line shows the results obtained by Edwards.

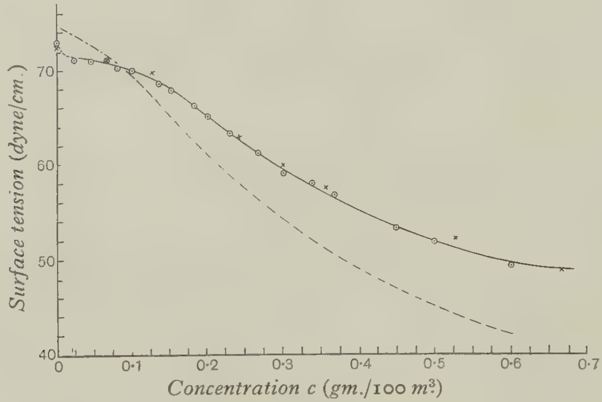


Fig. 2.

We have, then, three sets of values for the variation of surface tension with concentration for *p*-toluidine solutions—one set obtained by the drop-weight method, one obtained by the method originally described by Ferguson and Dowson, and the third set obtained by the variation of this method just described. The values in the first and third of these sets are in reasonable agreement, and it seems probable therefore that the results obtained by Edwards are due to some difference in the quality of the *p*-toluidine which he employed.

One or two side-issues of minor interest may be noted, arising from experiments made with a view to obtaining some knowledge of the variation of the surface tension of *p*-toluidine with time.

Two solutions were prepared, and kept in separate beakers. One beaker was kept covered, the other was left exposed to the atmosphere. The surface tensions were determined at approximately daily intervals. After a slight rise the surface tension of the liquid in the covered beaker remained constant over a period of 10 days.

In spite of steady evaporation (which was duly measured) from the open beaker and a very obvious fouling of the surface, the surface tension of this specimen rose fairly regularly from an original value of 51·1 dyne/cm. to a value of 71·0 dyne/cm. at the end of 33 days. Suspecting a possible change in the toluidine we applied the usual test* and found that the solution did not give the slightest response. We place

* Diazotization and addition of ferric chloride solution.

the fact on record; what chemical change had taken place we do not presume to say.

We noticed also that the solution was remarkably sensitive to fog and smoke, and might, indeed, be used as a nephometer. A day of heavy fog caused a sudden drop in the surface tension from 65·5 dyne/cm. to 63·7 dyne/cm., followed the day after by a recovery to 64·9 dyne/cm. Similarly an invasion of smoke from a chimney caused a drop of 6 dyne/cm. followed by an equal recovery.

To sum up: in the second part of this paper we describe a series of determinations of the surface tension of *p*-toluidine by a variant of a method previously described. The technique is very simple, and results may be obtained and worked out in a few minutes. The method possesses the advantages of the pressure method described in the paper by Ferguson and Dowson and, as with the method described in the first part of this paper, it may be made direct-reading inasmuch as the readings of the manometer have merely to be multiplied by a factor which for a given capillary is constant, in order to obtain the surface tension in dynes per centimetre.

And, it must be noted, this simple process includes the first-order meniscus corrections.

§ 3. ACKNOWLEDGMENT

These experiments were carried out in the laboratories of the East London College and our thanks are due to Prof. H. R. Robinson for the facilities which he placed at our disposal.

DISCUSSION

Mr R. C. Brown. The authors have evolved a method of measuring surface tensions of small quantities of liquid which is an undoubted advance on the older method of holding the capillary tube vertically, particularly as the necessity of knowing the density of the liquid is eliminated.

I was particularly interested in the results for para-toluidine solutions obtained by the second method described in the paper. As they remark, the curve which the authors obtain is in fair agreement with that of Gans and Harkins and those which I myself determined some time ago; but the agreement is only fair, and it seems to me that this must always be so. Para-toluidine is very capillary-active and is strongly adsorbed on to the liquid/air interface, and it is the surface concentration which determines the surface tension of the solution. The surface concentration is, of course, a function of the bulk concentration, but it seems reasonable that two separate observers should put exactly the same numbers of grams of solid into 100 cm^3 of solution and yet obtain somewhat different numbers of molecules per cm^2 of the surface film, which we can suppose is 1 molecule thick. The curve itself, when inverted, very closely resembles those obtained when the surface "pressure" of a monomolecular film is plotted against its area, and I suppose this is what we

have been doing in an indirect (and therefore less accurate) way, since the surface pressure of a film is the difference between its surface tension and that of pure water.

It is known that a film of camphor on a water surface will evaporate into the air. Might not the surface film of para-toluidine do the same thing? Thus the molecules of solid in an exposed surface of a solution would continually escape and be replaced by others coming from the bulk until, in time, the solution would become almost pure water. This would explain the failure of a 33-days-old solution to respond to a chemical test for para-toluidine.

Mr T. Smith. The authors have removed one measurement by rotating the capillary tube through a right angle from the vertical to the horizontal position, but have been led to adopt conditions in which the meniscus is distorted, and are limited to tubes of very small bore so that it may be negligible. A procedure which promises some advantages is to keep the tube vertical and use the plane surface criterion, but measure the pressures for the two vertical positions obtained by rotating the tube through two right angles about a horizontal axis. The equation for the surface tension then takes the very simple form

$$\gamma = \tfrac{1}{4} g r \rho_1 (h_1 + h_1') = \tfrac{1}{4} r (h_1 + h_1').$$

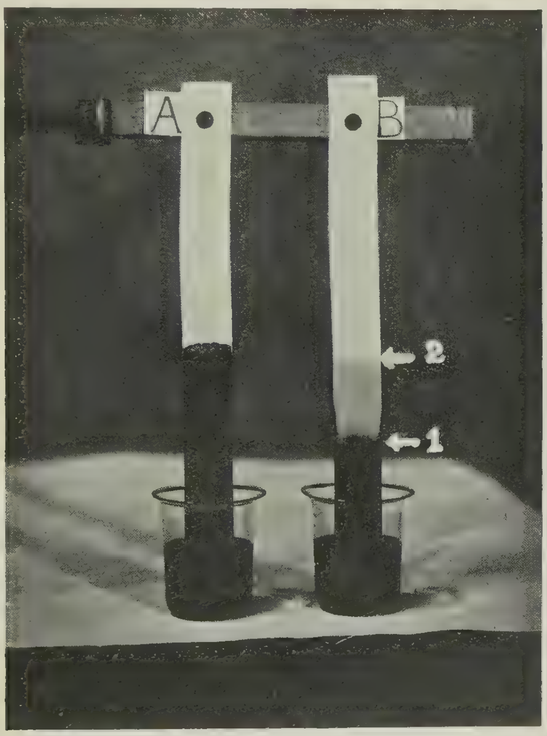
Authors' reply. We desire to thank Mr Brown for his very helpful remarks, particularly for those concerning the possible evaporation of *p*-toluidine from the surface film. The suggestion is a most useful one, and its possibilities should be explored.

Mr T. Smith's suggested method gives the required elimination, and very neatly. It must be remembered, however, that one of the difficulties encountered in the practice of the vertical-tube method with volatile liquids is due to the very rapid evaporation which takes place, causing a shortening of the liquid column. We have found it advisable to take the pressure reading immediately after measuring the length of the liquid column, and we fear that the time involved in the reversal and resetting may introduce an element of error, at least where volatile liquids are concerned. It is one of the advantages of the horizontal-tube method that this source of error does not enter into the problem. Another, but quite minor, advantage is the ease with which the method may be made direct-reading.

DEMONSTRATION

"Capillary adsorption due to surface tension." *Demonstration given on May* 20, 1932, *by* D. OWEN, B.A., D.Sc., F.Inst.P.

A solution of fuchsin in water, or ordinary red ink, has a surface tension slightly lower than that of pure water. When a vertical strip of filter paper is dipped into such a solution a red band advances up the paper, but immediately in front of it, constituting a vanguard of narrow depth, there is a colourless wet fringe. The fuchsin molecules thus show a preference for the walls of the capillaries, whilst the water molecules seek the axes and so advance further.



An aqueous solution of a salt, on the other hand, has a surface tension exceeding that of pure water. It was therefore anticipated that if the experiment were made with red ink or fuchsin solution to which some salt had been added, the fuchsin would be forced more strongly to the walls, whilst the salt molecules would tend to the centre of the capillaries. The coloured region would in this event be shortened, and the colourless region increased in length.

On trying the experiment these conclusions are found to be strikingly realized. In place of a narrow colourless band of about a millimetre the colourless band now extends over a length approaching that of the coloured region itself. This is shown in the photograph reproduced in the illustration, where *A*, representing the rise with red ink, shows an inconspicuous colourless fringe at the top; whilst *B*, representing the rise when the mixture of red ink and salt is used, shows the red band terminating at 1, and the colourless band extending from 1 to 2. The experiment works well with a solution of common salt of moderate strength.

REVIEWS OF BOOKS

Chaucer on the Astrolabe (*with the original illustrations*). Second and abbreviated edition, revised by R. T. GUNTHER. Pp. 96, 68 illustrations. (Oxford: Printed for the author, 5 Folly Bridge, Oxford.) 7*s*. 6*d*. net.

Dr Gunther, to whose painstaking researches on mediaeval instruments we owe so much, has put us deeper in his debt by issuing an edition of Chaucer's treatise at a price which puts the volume, even in these hard times, within the reach of most of us who are interested in such matters.

How comes it that even now Chaucer's merits as a man of science are not so fully recognized as they might be? Of the generous width and accuracy of his knowledge no one can doubt who has carefully read the treatise—compilation though it may be. Chaucer himself disarms criticism on this point when he says to his son, "I ne usurpe nat to have founde this werk of my labour or of myn engyn. I nam but a lewd compilatour of the labour of olde Astrologers, and have hit translated in myn English only for thy doctrine; and with this swerd shal I sleen envye." But there is yet more evidence of Chaucer's learning in the exact and careful references, especially to matters astronomical, which are scattered throughout his works, references which show him as a painstaking enquirer, well abreast of the science of his day. But despite this, and despite the fact that, till the twentieth century, Chaucer's was the standard English treatise on the use of this fascinating instrument, his name does not loom largely in the story of the development of physical science. His fame as poet overshadowed these lesser activities, and made it all the less likely that the historian of science should look to Chaucer for any serious contribution to his tale.

The treatise on the astrolabe is by no means difficult of access to-day. Standard one-volume editions of Chaucer's works usually include it. Skeat produced a scholarly and fully annotated edition as one of the publications of the Early English Text Society, and Dr Gunther issued an annotated edition in 1929 as volume 5 of *Early Science in Oxford*. Now we have the handy volume under review, which may be regarded as a second edition of the one just mentioned.

"Never the time and the place and the loved one all together"—so, at least, may we lament when we turn over the pages of the cheap editions. Those in the one-volume Chaucers are frankly pared to the bone. Illustrations, annotations, bibliographical details —all are gone, and the reader is left with the bare text. Skeat delighted us with a wealth of varied lore, but sorely cut down the number of plates.

Now Dr Gunther presents us with a text written in modernized form, and accompanied by all the illustrations which serve to give liveliness to Chaucer's explanations of the problems to be solved. But here he stops; we find practically no bibliographical details, and none of those discussions on side-issues which add so much to the interest for those of us who dearly love a digression.

Is it too much to ask that, when a third edition appears, Dr Gunther should write a score or so of pages by way of introduction to the work? Such an introduction would add little to the cost of the book, much to its already great value. A. F.

Les Principes de la Mécanique Quantique, by P. A. M. Dirac, translated by Al. Proca and J. Ullmo. Pp. viii + 314. (Paris: Les Presses Universitaires de France.) 95 fr.

When the tourist wishes to refer to a certain well-known work he calls it Baedeker, and not Baedeker's Guide-book, and in the same way one who wishes to refer to the volume which guides him round the subtler parts of the quantum theory calls it simply Dirac. Dirac has already been translated into German, and we now welcome the French translation. The translators, Messrs Proca and Ullmo, have themselves made good contributions to the subject and are eminently competent to carry out their task, and they have succeeded in reproducing to a great extent in the translation the lucid style of exposition of the original. There is a small addition by Mr Proca at the end, an account of the theory of Poisson brackets, which will prove useful especially to two classes which must now include nearly all physicists—the young who came straight to the new theory without having to labour at the classical, and the old who knew all about the brackets once, but have forgotten them in the struggle to understand the new developments. C. G. D.

An Outline of Wave Mechanics, by N. F. Mott. Pp. 155. (London: Cambridge University Press.) 8*s*. 6*d*.

The general plan of this book is less ambitious than that of many recent books on the subject. There is no effort to discuss questions of philosophy and metaphysics, and no advanced mathematics are used. In particular, Hamiltonian and Jacobian methods in dynamics are not once referred to in the whole of the book.

The word "outline" is included in the title and is an excellent description of the scope of the work. The subject-matter includes a treatment of the problems of the hydrogen molecule and the helium atom, including the explanation of the existence of non-combining terms, whilst there is also an account of the success of wave mechanics in explaining radioactive disintegration, and the concluding chapter of the book deals with electron spin.

A word which would equally well describe the contents is "survey." We pass rapidly over the main roads of this tract of country, in the company of a guide who is familiar with it; he points out the salient features of the landscape, and after enjoying his escort, most travellers will feel that they have noticed regions which they wish to explore in greater detail at some future time, either alone, or in the company of a pioneer who has already examined them. Nevertheless, we cannot yet say that the perfect book on wave mechanics has been written. When it appears, it will most certainly have many features in common with this one, but, of course, it will contain examples for the student-reader to cut his teeth on. We may also suppose that it will not contain the infelicities of expression which are noticeable with unpleasing frequency in this book.

"Suppose first that there *were* no interaction,...then each electron *will* move...."

"Let us consider what *will* happen if such an atom *were* placed in a magnetic field...."

"The pair may have *either* of the *three* values...."

Sentences such as these jar the nerves, although they do not in any way lessen the scientific value of the work in which they occur; the next, however, might well cause a certain puzzlement: "An electron shut up in a box could not have any energy, but could only exist in stationary states of energy E_1, E_2...." It means that an electron in a box could not have any arbitrary amount of energy, but could exist only in stationary states having energy E_1, E_2.

Having only one other criticism, and that an equally trivial one, the reviewer may as well dispose of it at this stage. It relates to the innovation by which a determinant is symbolized by placing two vertical lines on each side of the array of its elements. This use of two vertical lines on each side of an array of quantities has already been appropriated for

two different purposes, and it is a great pity to use it in a third way, when the ordinary determinant notation has for so long been well established.

Turning now from the differences between this book and the ideal, to consider the similarities, we find in the first place that the historical development of the subject is almost entirely disregarded. The author has considered wave mechanics as a whole, and has set out its ideas and methods in an order determined only by the logic of the processes, and not by the accidental chain of circumstances which led to their formulation. To take an example, we may refer to the treatment of the principle of indeterminacy. Instead of being invited to consider hypothetical experiments with γ-ray microscopes, the reader is led to expect the principle simply from an examination of the law of propagation of a wave packet, combined with the principles previously developed. The fact that the theory provides no wave packet capable of describing a beam of electrons with less than a certain dispersion in their generalized coordinates, together with the fact that it gives results which accord with experience, is taken to show that an electron beam with less than this minimum dispersion could not exist in nature. It follows that for a single electron there must be a similar dispersion in its possible coordinate-configurations; this is simply one way of stating the principle in question.

In a somewhat similar way the very foundations of wave mechanics are so developed that each step follows almost as a logical necessity from what precedes. One of the most elegant examples of this treatment is the section which leads up to the determination of the boundary conditions satisfied by a de Broglie wave.

The book will appeal not only to those who find the text-books of de Broglie and Temple useful, but also to that class of physicists who, though not primarily mathematical physicists, wish to understand the workings as well as the results of the new methods. Such a work was sorely needed, and one must be thankful to the author for filling the gap so well.

J. H. A.

Partial Differential Equations of Mathematical Physics, by H. BATEMAN, M.A., Ph.D. Pp. xxii + 522. (London: Cambridge University Press.) 42*s*.

This is a large volume of a little over 500 pages devoted to the consideration of a large number of problems occurring in physics. The title indicates that the problems are those in association with which a differential equation presents itself for solution. Such equations are of a very varied character and a choice has to be made in the compilation of even a large volume. The author chooses those which fall into a particular form which covers a surprisingly useful and wide range.

In text-books on physics it occurs frequently, on account of limitations of space and of the impossibility of turning a physics text-book into a mathematical one, that equations are solved incompletely and sometimes by methods that are mathematically unsound. This book is a work of reference in which these deficiencies are made good. One has only to glance through the book to realize how wide is the field of mathematics which the physicist must explore. The reader is usually anxious to hurry on from one region to another without too much attention to the connecting paths, so that an author has no easy task before him when he sets out to make a mathematical chart, which, while sufficiently complete, is not so detailed that it becomes irritating. In this respect we believe the author to be successful.

The problems are grouped in chapters under summary headings and an index of subjects and authors makes reference easy. The second chapter, for example, is entitled, "Applications of the Integral Theorems of Gauss and Stokes," and problems in hydrodynamics, elasticity and heat with many others are associated under this heading.

The general method is to assume as a starting-point results which are directly to be obtained from works on physics and to proceed to the solution, overlapping being thus avoided: for example in a problem on the bending of a beam, the expressions for

shearing-force and bending-moment form the starting-points and we then have a purely mathematical discussion of equations resulting in particular problems. This is exactly what the physicist, at any rate, requires.

This problem also illustrates a case where boundary conditions have to be considered. The author discusses various types of boundary conditions in the first chapter—a very useful discussion for physicists in general, and for those in particular who are interested in the new developments, where boundary conditions are of great importance.

The author does not consider the differential equations of the new quantum theory, but the new theory, in some of its aspects, makes so much use of classical methods that the student of recent advances in atomic physics will find useful material here.

Chapter IX, on paraboloidal coordinates, introduces Hermite's polynomial, and examples of orthogonal functions are scattered about the book. A number of books have been written in order to assist the physicist in the mathematics appropriate to his subject, especially by continental writers. This work in English is welcomed particularly on account of its originality and usefulness, but also on account of the unity introduced into a subject which might easily have become a tedious collection of isolated problems.

H. T. F.

Solutions Superficielles Fluides à deux Dimensions et Stratifications Monomoléculaires, by ANDRÉ MARCELIN. Pp. 163 with 86 figures including 8 coloured plates. (Paris: Les Presses Universitaires de France.) 80 fr.

The term "surface solution" has come to be the recognized physical name for molecules of a substance in the surface of a liquid, which, by their presence, alter the surface tension of the liquid. The study of such two-dimensional solutions has in some ways proved more fundamental than the study of bulk solutions. For instance, the "pressure" of a layer of molecules can be directly measured by the force on 1 cm. of a floating barrier separating the layer from a free surface of the solvent, whereas osmotic pressure-determinations are apt to be clumsy and their results difficult to interpret.

After a short introductory chapter M. Marcelin gives us the history of the surface film, beginning with divers observations on the calming of the sea by oil which suggested that possibly a film whose thickness was comparable with that of a single molecule would produce the desired effect. The author then describes the experiments of Miss Pockels, Lord Rayleigh, Devaux and himself, and gives the values of so-called molecular diameters calculated therefrom. This is followed by an outline of Langmuir's orientation theory.

A large section of the book is devoted to the refined experiments of Langmuir, Adam and the author himself, which led to the establishment of the perfect-gas relation connecting the surface pressure of a film with its area and temperature, provided the film is unsaturated. The absorption equation of Gibbs is treated briefly both from the classical and modern points of view.

No less interesting than the foregoing is the account of the experiments of Guyot on the lowering of the Volta effect between an electrolyte and a metal by the presence of a surface film. The results of this investigation give us values for dipole moments and the average angle which the axis of the molecule makes with the plane of the film.

Finally M. Marcelin gives accounts of optical investigations of the black spots of soap films and the thickness of paratoluidine crystals formed both from solution and by sublimation. Several very good colour-reproductions are given.

The whole volume is full of interest and should bring home to the reader the fact that in the study of thin films we have a powerful means of obtaining insight into molecular behaviour.

Every important experiment that is mentioned is clearly described, but should the reader desire further detail he will find the extensive bibliography of great value.

R. C. B.

The Adsorption of Gases by Solids. A General Discussion held by the Faraday Society, January 1932. Pp. 448. (London: Gurney and Jackson.) 15*s.*

The contents of this volume, including summaries of work already published and also new experimental and theoretical papers, cover a wide field. Much discussion centred about the reality of two types of adsorption, one of which is small and in evidence at low temperatures and involves low binding energy; while the second type is larger and in evidence at higher temperatures, involves large heats of adsorption, and proceeds with measurable velocity. The second type, called "activated adsorption," or "chemisorption," is considered by some contributors as due to the removal of adsorbed films of foreign material with increasing temperature. New experimental methods include the great increase which is effected in the electron-emissivity of tungsten by small adsorbed amounts of electropositive metals, the study of electron-diffraction from surfaces on which there is adsorbed gas, and the transformation of ortho- into para-hydrogen on surfaces. On the whole, the experimental side is less prominent than the theoretical, and in the latter section there are papers of considerable interest in which the effects of van der Waals's forces, now known from quantum mechanics to be inversely as the seventh power of the distance, and quantum valency forces arising from the interaction of electrons, are considered. The effects of electrical forces due to dipoles are discussed also, although in one communication it is claimed that these are usually negligible. The interaction between an inert-gas atom and a metal, the latter considered as containing a perfectly polarizable electric continuum, and between a gas atom and an ionic crystal, have been worked out. An interesting discussion as to a possible explanation of activated adsorption starts from the supposed dissociation of the adsorbed gas molecule when the cohesion between its constituent atoms and the adsorbent is greater than the dissociation energy. The approach of atoms to the metal surface gives rise to irregularities with new conduction-electron levels in the metal atoms, and this may explain activated adsorption at higher temperatures. There is much experimental evidence that adsorbed particles may be in a mobile state and capable of moving about over the surface as a kind of two-dimensional liquid. Their penetration into microscopic cracks in the solid, caused by unbalanced surface forces, will explain absorption into the interior of the solid, which may also be activated. Many other aspects of the subject are dealt with, and the volume provides a useful survey of recent advances in the subject, many of direct interest to physicists. J. R. P.

Manual of Meteorology, Volume 4. *Meteorological Calculus: Pressure and Wind*, by Sir NAPIER SHAW, LL.D., Sc.D., F.R.S., with the assistance of ELAINE AUSTIN, M.A. Pp. xx + 359. (London: Cambridge University Press.) 30*s.*

The issue of this volume completes a manual which a former number of these *Proceedings* anticipated "would probably be for some time the standard work on meteorology." In the dedication of it to the memory of W. H. Dines, F.R.S., it is called a "volume of reminiscences," but half of it was issued for official use in the Meteorological Office in 1918 and to the public a year later. The 160 pages of this earlier issue have now been expanded to 195 pages by the addition of new matter such as the section on "peculiarities of surface isobars." Two new chapters on the laws of atmospheric motion and on the general equations of motion of a parcel of air serve as an introduction to the chapters of the previous issue, and the volume closes with two further new chapters, one on hypotheses and realities as to low- and high-pressure areas and the other entitled "retrospective and prospective." In the preface Sir Napier Shaw takes credit to himself for the "discursiveness" which has been charged against volume 3 and claims that volume 4 has the same characteristic. With regard to the last two chapters most readers will admit his claim, but they will feel some doubt as to the earlier ones. The last chapters constitute a summary of

the authors' views, which depart considerably from those generally held. They regard the horizontal motion of the wind as the independent variable and the pressure as the dependent. Velocity and entropy are regarded as the two quantities which determine all atmospheric phenomena. Apart from these innovations the two chapters present the views of the Norwegian meteorologists, but the presentation is not always as clear as it might be and terms are introduced which are not in common use, e.g. "syncopated by half a period" for delayed by half a period (p. 331), "underworld" for the lower air (p. 335), "if the earth went dry" printed as a quotation but having no reference to prohibition (p. 333). An index of 11 pages is provided and the volume ends with 10 pages entitled, "The Drama of the Atmosphere," and a list of the "dramatis personae"—that is, the contents of the whole manual.

The printing and general appearance of the volume are excellent. C. H. L.

Wireless Receivers: The Principles of their Design, by C. W. OATLEY, M.A., M.Sc. Pp. vii + 103. (London: Methuen & Co., Ltd.) 2*s*. 6*d*.

It would be impossible to do full justice to this title in the space of one of Messrs Methuen's "Monographs on Physical Subjects," to which series this little book belongs. Mr Oatley has, however, made good use of the space at his disposal to produce what should prove to be a useful introduction to a popular subject.

The book is divided into chapters dealing with the various "stages" into which a wireless receiver may, in general, be divided—namely, the aerial and tuner system, h.f. amplifier, detector, l.f. amplifier, and power stage; the fundamental functions of each stage and the way in which incorrect design may affect operation being explained in precise mathematical language. A bibliography is appended, which serves to amplify the text considerably and especially to draw the reader's attention to the many recent advances in the design of highly selective broadcast receivers; comprising some fifty items, this cannot be considered as unduly bulky.

Two special classes of receiver do not receive mention in the text. One of these, the superheterodyne, claims a special section of the bibliography, and the other, the super-regenerative, although likely to become of importance with increasing interest in ultra-high-frequency telephony, is of such limited application at the moment that omission of its description is scarcely surprising.

The average student of physics has but a vague knowledge of wireless principles and of thermionic valve circuits; this little book can be recommended as offering an excellent means of placing such knowledge on a sounder, quantitative basis. R. A. F.

Magnetism and Electricity (second edition), by E. NIGHTINGALE, M.Sc. Pp. xvi + 294. (London: G. Bell and Sons, Ltd.) 4*s*. 6*d*.

The author, who has already produced a very valuable elementary book on sound, has now issued the second edition of this *Electricity and Magnetism*, which is of School Certificate standard.

Great attention is devoted to the historical development of the subject, with a wealth of illustration in the way of photographs and small but clear diagrams, which almost average one per page. The mathematical treatment is not advanced, but numbers of examples are included at the ends of the chapters. Emphasis is also laid on the practical applications of the principles described. Frictional electricity is treated, probably with good reason, on conventional lines, and the electron is only mentioned towards the end of the book. Credit is fairly given to Cavendish for the discovery of Ohm's law by means of a condenser-discharge through his own body. Other interesting matters include electric cardiographs, electric ovens, and maps showing the cable systems of the world and the "main grid" transmission lines of England. The book is indeed of remarkably good value and printing and production are all that could be desired. J. E. C.

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